Definition: A normed operator ideal is an assignement such that

1) For every pair of Banach spaces X, and Y, a(X,Y) a normed Banach space which contains the finite rank operators
2) $a(x^*\otimes y) = \|x^*\| \|y\|$ for all $x^* \in X^*$ $y \in Y$
3) For all $S: X_1 \to X$ $R: Y \to Y_1$
$$\alpha(RTS) \le \|R\| \alpha(T) \|S\|$$

Theorem: L(X,Y) is the largest and N(X,Y) is the smallest normed operator ideal.

Proof: We have to show that
$\|T\| \le a(T) \le N(T)$
for all T. Indeed, let x in X and y* in Y*. Note that a(K,K)=K.
Then we define S:K to X by S(t)=tx and R(y)= y*(y) from Y to K.
It follows that
$|y^*(Tx)| \le a(RTS) \le \|R\| a(T) \|S\| = \|y^*\| \|x\| a(R)$.
Let us recall that
$N(T)= \inf \{ \sum \|x_k^*\| \|y_k\| \mid T(x) = \sum_k x_k^*(x) y_k \}$
Using a Cauchy sequence argument, we deduce a(T) N(T) from ii and the triangle inequality.

Remark: A tensor norm α on the couples (X*,Y) allows us to define and ideal norm via,
$a(T) = \alpha(\sum x_k^* \otimes y_k)$ $T = \sum x_k^* \otimes y_k$
Conversely, the restriction of an ideal norm to couples (X*,Y) is a tensor norm (with restricted domain).

Duality: For an ideal norm we may define
$a^*(S) = \sup \{ |tr(ST)| : a(T) \le 1 \}$
Here the supremum is taken over finite rank maps.

Remark: 1) This is not the only way to define a dual operation.
2) With this assignement
α(X*,Y)* = $\underset{\alpha}{a^*}$(Y,X) .

Examples: 1) L(X,Y)
2) K(X,Y), the compact operators with the operator norm.
3) The closure of the finite rank maps with respect to any operator ideal norm, in particular the operator norm.
4) p-summing map,
5) a*(X,Y)
6) p-Integral maps.
7) N_p(X,Y) the set of bounded linear maps such that
v_p(T)= inf $\left(\sum \|x_k^*\|_{X^*}^p\right)^{1/p} w_p(y_k)$

$$T(x) = \sum x_k^*(x) y_k$$

is finite