Banach spaces-Homework 4

Due date: Monday, February 25

1. Show that $\pi_2(id : \ell_1^n \to \ell_2^n) = 1$ holds in the real case (also true complex).

2. Let X, Y be Banach spaces and G be compact group, and a representations $\alpha : G \to B(X)$, $\beta : G \to B(Y)$ such that

$$\|\alpha(g)(x)\|_X \;=\; \|x\|$$

for all $x \in X$ and

$$\|\beta(g)(y)\|_Y \;=\; \|y\|$$

and $y \in Y$. Now let $T : X \to Y$ be p-summing. Show that there exists a G-invariant measure μ on B_{X^*} such that

$$\|Tx\|_Y^p \;\le\; \pi_p(T)^p \int |x^*(x)|^p d\mu(x^*)\ .$$

Here G invariant means

$$\int f(\alpha_g^*(x^*))d\mu(x^*) \;=\; \int f(x^*)d\mu(x^*)$$

for every measurable f.

3. Let us call a measure μ on the unit sphere of $\ell_{p'}^n$ special if μ is invariant under permutations and changes of signs. Describe all the special measures on the extreme points of ℓ_1^n (i.e. $p = \infty$).

4. Apply the previous problems and show that $\pi_p(id : \ell_s^n \to \ell_q^n)$ is smallest constant C such that

$$\|x\|_q^p \;\le\; C^p \int_{B_{s'}^n} |x^*(x)|^p d\mu(x^*)$$

holds for special probability measures.