Homework 5

Due Date: Monday October 9. Submission in pairs possible.

1. Show that
   \[ \pi_1(id_{\ell^p_n}) \geq \sqrt{n} . \]

2. Show that
   \[ \pi_1(RS) \leq \pi_2(R) \pi_2(S) . \]

3. A linear map \(T : X \to Y\) is called \((p, q)\) summing if there is a constant \(C > 0\) such that
   \[ \left( \sum_k \|Tx_k\|_Y^p \right)^{\frac{1}{p}} \leq C \sup_{\|x^*\| \leq 1} \left( \sum_k |x^*(x_k)|^q \right)^{\frac{1}{q}} . \]
   Then \(\pi_{p,q}(T) = \inf C\) where the infimum is taken over \(C\) such that the above inequality holds for all finite sequences in \(X\). Show that for \(\frac{1}{p} - \frac{1}{q_1} = \frac{1}{p_2} - \frac{1}{q_2}\) and \(p_2 \leq p_1\) one has
   \[ \pi_{p_2,q_2}(T) \leq \pi_{p_1,q_1}(T) . \]
   (Hint: Use equality in Hölder’s inequality).

4. Show that \(T\) is absolutely one summing iff
   \[ \sum_k \varepsilon_k x_k \]
   converges for all \(\varepsilon_k = \omega \) implies
   \[ \sum_k \|T(x_k)\| \]
   is finite. For one implication you may use the fact that
   \[ \sum_k \varepsilon_k x_k \]
   finite for \(\varepsilon_k = \pm 1\) implies
   \[ \sup_{\varepsilon_k = \pm 1} \left\| \sum_k \varepsilon_k x_k \right\| < \infty . \]