Homework 7

Due: Friday, April 24

1. Find two Banach algebras $1 \in A \subset B$ and $x \in A$ such that

$$\sigma_A(x) \neq \sigma_B(x) \, .$$

2. Let $A \in M_d(\mathbb{C})$ and $X_\lambda = \{x \in \ell_2^d | Ax = \lambda x\}$. Use holomorphic calculus to who that there is a projection P onto X_λ.

3. Let A be a $d \times d$ matrix with $\lambda_1, ..., \lambda_d$ distinct eigenvalues. Calculate the spectrum.

4. Find a $d \times d$ matrix where the spectrum does not equal the eigenvalues.

5. What is the spectrum of a nilpotent matrix.

6. Let $A = B(\ell_1)$ and $x(e_k) = \lambda_k e_k$ be a diagonal operator. Determine the spectrum.

7. Let $A = B(\ell_1^n)$ and A be the matrix with $A_{ij} = 1$ for $i \geq j$, 0 else. Find the spectrum.