(1) The gradient descent algorithm starts with $x_0$ and produces a sequence $x_{n+1} = \phi(x_n)$ of vectors in $\mathbb{R}^d$ such that

$$d(x_n, x_{n+1}) \leq 4/5d(x_n, x_{n-1})$$

Show that the algorithm converges.

(2) Let $(x_n)$ be a bounded sequence in $\mathbb{R}^d$ such that $\lim((x_n))$ has only one point. Show that $(x_n)$ is convergent.

(3) Find a metric space $(X, d)$ and sequence $(x_n)$ such that $\lim((x_n))$ has only one point and the sequence is not converging.

(4) Show that the finite intersection of open sets in a metric space are open. Give an example that this fails for a countable intersection.

(5) Show that an infinite intersection of closed sets is closed. Find an example of a countable union of closed sets which is no longer closed.

(6) 14.8 p102

(7) 14.6 p102

(8) 13.7 page 93