(1) The gradient descent algorithm startys with x_0 and produces a sequence $x_{n+1} = \phi(x_n)$ of vectors in $\mathbb{R}^d$ such that

$$d(x_n, x_{n+1}) \leq 4/5 d(x_n, x_{n-1})$$

Show that the algorithm converges.

(2) Let (x_n) be a bounded sequence in $\mathbb{R}^d$ such that $\mathrm{Lim}((x_n))$ has only one point. Show that (x_n) is convergent.

(3) Find a metric space (X, d) and sequence (x_n) such that $\mathrm{Lim}((x_n))$ has only one point and the sequence is not converging.

(4) Show that the finite intersection of open sets in a metric space are open. Give an example that this fails for a countable intersection.

(5) Show that an infinite intersection of closed sets is closet. Find an example of a countable union of closed sets which is no longer closed.

(6) 14.8 p102

(7) 14.6 p102

(8) 13.7 page 93