

Take Home Final

Name: Submission by May 5, 11.59 pm. Use PDF format and send to mjunge@illinois.edu

We consider the differential equation

$$f'(x) = xf(x) + x^2, f(0) = a_0.$$

- i) Use power series to solve the
- ii) Determine the radius of convergence for arbitrary initial value **Hint:** Try $R = \infty$.
- iii) Show that the set

$$S = \{f' | f(x) = xf(x) + x^2, f(0) \in [-1, 1]\} \subset C[-1, 1]$$

is connected **Hint:** You can write a solution in the form $f = f_1 + f(0)f_2$ for some nice functions f_1, f_2 , why?) .

- iv) Show that the closure of the set S above is compact.
- v) Is S closed?

Please, take a little time to justify your arguments and mention important results you use in your argument.