Archimedes

Archimedes of Syracuse (287-212 B.C.) was the greatest mathematician in ancient times.

Life: Known for
  mechanical inventions:
    planetarium predict eclipses of sun and moon,
    water pumping screw,
    pulley devices
    "give me a place to stand and I can move the earth"
  war machines
  feared by Romans,
although Archimedes died in besiege of Syracuse (some kind of accident)

for Archimedes: diversions of geometry at play. Only his written work on mathematics is known today.

About mathematical work:

  mathematical elegance
  model of mathematical exposition
  gradually unfolding plan of attack
  elimination of irrelevant data
  no clue where idea comes from-awe
  clarity
We will discuss
Measurement of Circle
Quadrature of Parabola
On the sphere and cylinder
Conoids and Pheroids

Archimedes was very proud on his work on sphere, it was to be mentioned on his tombstone.

Comments on the method of exposition in history of mathematics

We use algebra to simplify assertions. $A=\pi dh$ was expressed as the area of a circle with radius the mean proportional between height and diameter.

Here we will go even further and use fractions in our calculations, in order, hopefully, to unravel the basic mathematical ideas. There are texts which are more faithful to the original style.
Circle

Before Archimedes it was known that the area of a circle was proportional to the diameter square, and the circumference was proportional to the diameter.

**Theorem:** (Archimedes) Let \( r \) be the radius and \( C \) be the circumference. Then the area satisfies

\[ A = \frac{1}{2} r C \]

Later Archimedes revealed geometric intuition:

The triangle \( \triangle \) and \( \bigcirc \) have the same area.

**Remark:** This was used to approximate \( \pi \).
Proof: Let us assume \( a(D) > a(\Delta) \).

Then we can find a polygon inscribed in \( D \) such that

\[ P_n \cap D \quad a(P_n) > a(\Delta) \]

Align the small triangles all parallel.

Then

\[ \text{area}(\Delta') \leq \text{area}(\Delta) < \text{area}(P_n) = \text{area}(\Delta_n) \]
Now assume $a(D) < a(\Delta)$

Then we can find a regular polygon s.t.
$D \leq Q_n \quad a(Q_n) < a(\Delta)$

We realign the small triangles

The height is the radius and length is perimeter of $Q_n$. Then

$a(D) \leq a(Q_n) < a(\Delta) \iff$ longer circuit
Archimedes inscribed a hexagon and then successively doubled the points as to find a good approximation of pi and the circumference. For this he developed an iteration formula for the circumference of the outside and inside polygon.

Outside:

\[ t_{2n} = \frac{t_n}{1 + \left(1 + t_n^2\right)^{1/2}} \]

Since OAD and PAC are similar and \(|PO|=|OC|\) we get

\[ \frac{|AD|}{|AO|} = \frac{|AC|}{|AO|+|OP|} = \frac{|AC|}{|AO|+|OC|} \]
Similarly for inside:

Then ABD bisects the angle ABC, and the triangles ABD, BPD and APC are similar.

\[
\frac{|AB|}{|AD|} = \frac{|BP|}{|BD|}, \quad \frac{|AC|}{|AD|} = \frac{|PC|}{|BD|}.
\]

\[
\frac{|AB| + |AC|}{|AD|} = \frac{|BP| + |PC|}{|BD|} = \frac{|BC|}{|BD|}.
\]

\[
\frac{S_n}{S_{2n}} = \frac{2 + \sqrt{4 - S_n^2}}{\sqrt{4 - S_{2n}^2}}.
\]
This gives \[ s_{2n}^2 = \frac{s_n^2}{2 + 14 - s_n^2} \]

**Theorem** \[ \frac{n}{2} s_n < \pi < n t_n \]

**Proof:** The first inequality follows from \[ n s_n = |D P_n| \leq |D D'| = 2 \pi \]

The second follows from \[ a(D) = \pi < a(\square_n) = n \frac{2t_n - 1}{2} = n t_n. \]