Johannes Kepler - 1571-1630

Kepler made a major discovery about planetary motions using infinitely divisible numbers:

I) Planet moves along elliptical orbit and the sun is in one of the focal points.

II) The radius sweeps out a constant area per time.

III) The square of period equals a constant times the cube of the major axis.
I) is derived using two compensating errors!

- By observation Kepler observed that the velocity at the points with largest and biggest distance from the sun is inverse proportional to the distance.

\[ v_2 \cdot t_2 = K = v_\Lambda \cdot r_\Lambda \]
- Kepler assumes that this remains true for all points on the orbit of the planet, i.e.

\[
v = \frac{d}{r}
\]

Here \( r \) is the distance to sun.

- Following what he thinks is Archimedes path, Kepler divides the circle in equal parts \( \Delta s = \frac{\text{Period}}{n} \)
• Then

\[ t = \sum_i \Delta t_i = \sum_i \frac{\Delta s}{v_i} = \frac{1}{k} \sum_i r_i \Delta s. \]

By analogy with the circle and approximation by triangles we have

\[ \sum_i r_i \Delta s = \frac{1}{2} \text{Area(segment)} \]

• Conclusion: Time passed is proportional to area swept out.
• Mistake (1) is not true, polar co-
    ordinates give

\[ A = \int_{\theta_0}^{\theta_1} \int_{0}^{r(\theta)} r \, dr \, d\theta = \frac{1}{2} \int_{\theta_0}^{\theta_1} r^2(\theta) \, d\theta. \]

Analogy with circle is not helpful.

• Further mathematical achievement

90 calculations for volumes of vine barrels and related shapes using
infinite divisibles, see page 103

\textit{Book: Stereometria}
• Kepler reproved Pappus theorem

\[ V = \pi a^2 (2\pi b). \]