Vector functions

$\gamma : [a,b] \longrightarrow \mathbb{R}^n \qquad (n = 2, 3)$

Recall $f : [a,b] \to \mathbb{R}$, $t \in [a,b]$

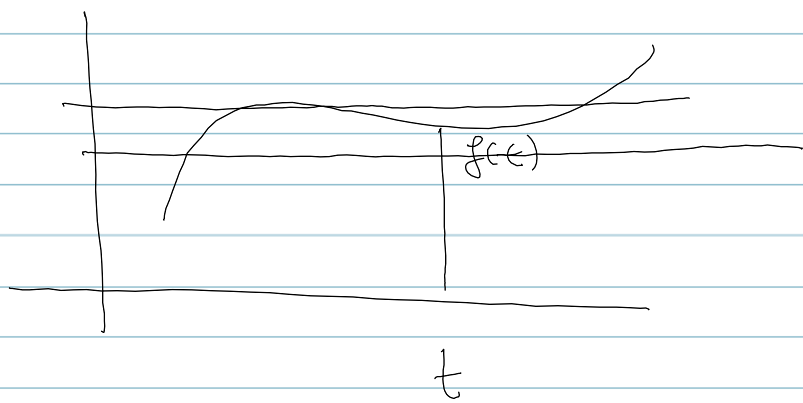
f ct at t if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that

continuous

$$|t - s| < \delta \implies |f(t) - f(s)| < \varepsilon$$

($\varepsilon - \delta$ Definition of continuity)

(equivalently $\boxed{\lim_{s \to t} f(s) = f(t)}$)



} ε - band around $f(t)$

Output precision is ε

Require input precision δ

Definition $\gamma : [a,b] \to \mathbb{R}^n$ $t \in [a,b]$ γ ct at t if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that

$$|t - s| < \delta \implies |\gamma(t) - \gamma(s)| < \varepsilon$$

Recall $|v| = \sqrt{v_1^2 + \cdots + v_n^2}$.

Calculus I, II $\longrightarrow$ Calculus III

A vector function is ct at t if all the coordin fucl are ct at

is diff'le at t — " — are diff

is 2diff at t if — " — are 2diff.

Example:

$$\gamma(t) = r(f(t)) \qquad f\,[a,b] \to [c,d]$$

Chain rule:

$$\gamma'(t) = r'(f(t))\, f'(t)$$

$$\gamma'(t) = \begin{pmatrix} \gamma_1'(t) \\ \vdots \\ \gamma_n'(t) \end{pmatrix}$$

vector of derivatives

"velocity vector"

<u>Theorem</u> a $\quad f_a : \mathbb{R}^2 \to \mathbb{R}^2 \quad f_a(v) = av$

New basis $\{v, w\}$. Then the matrix $\tilde{a}$ of f_a with respect to the new basis is given by

$$\tilde{a} = c^{-1} a c$$

Where $c = (v, w)$ is the unique matrix such that $c\binom{1}{0} = v \quad c\binom{0}{1} = w$.

<u>Proof</u> the matrix of $\tilde{a}$

$$f_a(v) = \lambda v + \mu w \qquad \tilde{a} = \begin{bmatrix} \lambda & \gamma \\ \mu & \delta \end{bmatrix}$$
$$f_a(w) = \gamma v + \delta w$$

Recall 1) $f_a(v) = av$ $\quad$ 2) $c^{-1} a v = \binom{\lambda}{\mu}$

$c^{-1} a w = \binom{\gamma}{\delta}$

$$c^{-1} a \overbrace{\begin{pmatrix} v & w \end{pmatrix}}^{c} = c^{-1} a c \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{bc } \; c\binom{1}{0} = v, \; c\binom{0}{1} = w \; ?$$

$$\begin{pmatrix} \lambda & \gamma \\ \mu & \delta \end{pmatrix} \qquad\qquad c^{-1} a c$$

(the first equal by ‖, the second by ‖)

$\boxed{\Sigma}$