Vector Functions

\[ \gamma : [a, b]^n \rightarrow \mathbb{R}^n \quad (n = 2, 3) \]

Recall \( f : [a, b] \rightarrow \mathbb{R}, \, t \in [a, b] \)

\( f \) is \( \epsilon \)-continuous at \( t \) if for every \( \epsilon > 0 \) there exists a \( \delta > 0 \) such that

\[ |t - s| < \delta \implies |f(t) - f(s)| < \epsilon \]

(\( \epsilon - \delta \) definition of continuity)

(equivalently \( \frac{\text{lim}}{s \rightarrow t} f(s) = f(t) \))

\[ \text{Output precision is} \quad \epsilon \\
\text{Require input precision} \quad \delta \]

Defining \( \gamma : [a, b]^n \rightarrow \mathbb{R}^n \) \( t \in [a, b] \), \( \gamma \) is \( \epsilon \)-continuous if

for every \( \epsilon > 0 \) there exists a \( \delta > 0 \) such that

\[ |t - s| < \delta \implies |\gamma(t) - \gamma(s)| < \epsilon \]

Recall \( \|v\| = \sqrt{v_1^2 + \ldots + v_n^2} \).
Calculus I, II → Calculus III

A function is $C^2$ at $t$ if all the second-order derivatives are continuous at $t$. The function is $C^2$ at $t$ if

$$f''(t)$$

are continuous at $t$.

Example:

$$y(t) = r \left( \frac{f(t)}{g(t)} \right) \quad \mathbb{R}^{0.5J} \rightarrow \mathbb{R}^{pJ}$$

Chain rule:

$$y'(t) = r' \left( \frac{f(t)}{g(t)} \right) \frac{f'(t)}{g'(t)}$$

$$y'(t) = \begin{pmatrix} y'_1(t) \\ y'_2(t) \end{pmatrix} \quad \text{vector of derivatives}$$

$$v(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} \quad \text{velocity vector}$$
Theorem \[ f_a : \mathbb{R}^2 \to \mathbb{R}^2 \quad f_a(v) = av \]

New basis \( v, w \). Then the matrix \( \tilde{a} \)
of \( f_a \) with respect to the new basis is given by

\[ \tilde{a} = c^{-1}ac \]

Where \( c = (v, w) \) is the unique matrix such that \( c(v) = \tilde{v}, c(w) = \tilde{w} \).

Proof The matrix of \( \tilde{a} \)

\[ f_a(v) = f(v) + \mu w \]
\[ f_a(w) = f(v) + \nu w \]

Recall 1) \( f_a(v) = av \) 2) \( ac^t av = (\mu) \)

\[ c^{-1}aw = (\gamma) \]

\[ c^{-1}a(v, w) = c^{-1}aC(1, 0) \quad bc \quad c(1) = v \]
\[ c(0) = w \]

\[ \begin{pmatrix} \mu & \gamma \\ \nu & \delta \end{pmatrix} = c^{-1}aC \]