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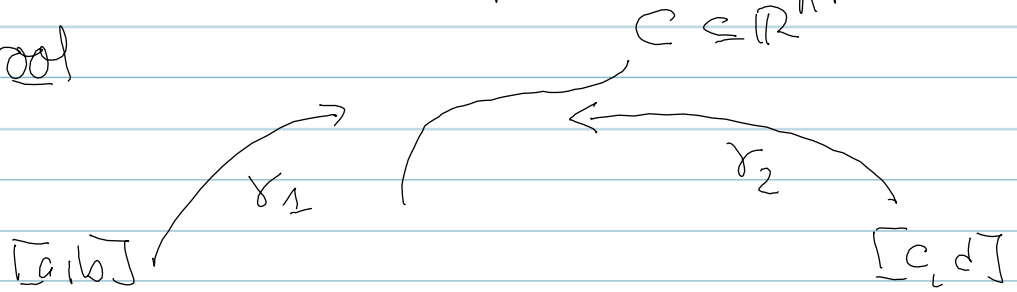
Theorem The arc length $L(\gamma) = \int_a^b |\gamma'(t)|\, dt$

does not depend on the parametrization.

Proof



Parametrization $s \neq t \Rightarrow \gamma_1(s) \neq \gamma_1(t)$

$s' \neq t' \Rightarrow \gamma_2(s') \neq \gamma_2(t')$

Reparametrization comes from true:

$t \in [a,b] \longrightarrow \gamma_1(t) \in C$ — unique

$\gamma_2(\tilde{t}) = \gamma_1(t)$

$\tilde{t} \in [c,d]$

Define $f(t) = \tilde{t}$ $\Rightarrow f$ diff

Find $\gamma_1(t) = \gamma_2(f(t))$ Hyster γ_1, γ_2 diff

$\gamma_1'(t) = \gamma_2'(f(t))\, f'(t)$

$\int_a^b |\gamma_1'(t)|\, dt = \int_a^b |\gamma_2'(f(t))|\, |f'(t)|\, dt$ → always the same sign automatic

$= \int_c^d |\gamma_2'(s)|\, ds$

change of var

□

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