

④ We multiply a matrix with vector as follows

$$\begin{matrix}
 n \times d \\
 \left[\begin{array}{cc}
 a_{11} & a_{1d} \\
 \vdots & \vdots \\
 a_{n1} & a_{nd}
 \end{array} \right]
 \begin{bmatrix}
 x_1 \\
 \vdots \\
 x_d
 \end{bmatrix}
 \end{matrix}$$

$$= \begin{bmatrix}
 a_{11}x_1 + \dots + a_{1d}x_d \\
 a_{21}x_1 + \dots + a_{2d}x_d \\
 \vdots \\
 a_{n1}x_1 + \dots + a_{nd}x_d
 \end{bmatrix} = y$$

Short $ax = y$ is a vector in \mathbb{R}^n

⑤ a $n \times d$ b $d \times m$

$$b = \begin{bmatrix}
 b_{11} & \dots & b_{1m} \\
 \vdots & & \vdots \\
 b_{d1} & \dots & b_{dm}
 \end{bmatrix}
 \quad
 b_j = \begin{bmatrix}
 b_{j1} \\
 \vdots \\
 b_{jd}
 \end{bmatrix}$$

$$ab = \begin{bmatrix}
 a(b_{11}) & \dots & a(b_{1m}) \\
 \vdots & & \vdots \\
 a(b_{d1}) & \dots & a(b_{dm})
 \end{bmatrix}$$

⑥ $a(b_j) = j$ the column.