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Welcome to

CALCULUS III

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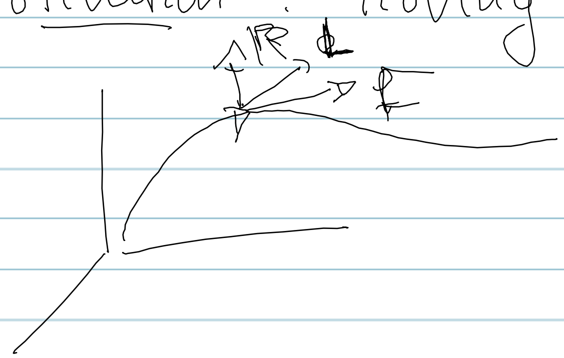
Altgeld 363

Previously Calculus was about

- Differentiation
- Integration
- Limits
- function in one variable

Now: Differentiation and Integration of functions of several variables

Motivation: Moving particle

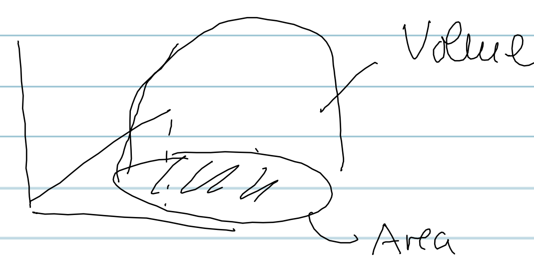


Moving frame on roller coaster

F forward

L left R right.

or



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Plan

1) Vectors in 2D

2) Vectors in 3D

3) Differentiation

4) Integration

5) Combining the two
Fundamental Theorem
of calculus in disguise

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Notation

$\mathbb{R}$ real line $\left(e, \sqrt{2}, \frac{p}{q}, k \in \right)$

$\mathbb{N}$ natural numbers ($0 \in \mathbb{N}$ or not ?)
1, 2, 3, 4 ....

$\mathbb{Z}$ integers $0, \pm k, \; k \in \mathbb{N}$

A set = collection of objects

A, B set

$$A \times B = \{(a,b) : a \in A, b \in B\}$$

read as "the set of all pairs (a,b) such that a is in A, b is in B

$a \in A$ a is an element of A.

Example $A = \{x \in \mathbb{R} : x \geq 0\}$

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(x,y) : x \in \mathbb{R}, y \in \mathbb{R}\}$$

$$\mathbb{R}^d = \underbrace{\mathbb{R} \times \cdots \times \mathbb{R}}_{d \text{ times}} = \{(x_1, \dots, x_d) : x_1 \in \mathbb{R}, \dots, x_d \in \mathbb{R}\}.$$

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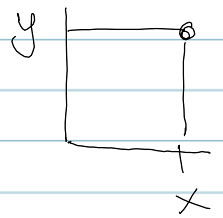
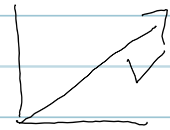
Vectors versus points

(moral distinction)

$\mathbb{R}^2 = \{(x,y) : x \in \mathbb{R}, y \in \mathbb{R}\}$





$((0,0),(x,y))$

Physics: $p = (x,y)$ is a point or location.

v is the direction of going from origin to point.

vectors correspond to velocity or force

Remark one can add vectors

Definition: In a vector space (such as $\mathbb{R}^2$) one can vectors and stretch vectors

$$\lambda v + \mu w = \lambda \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \mu \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$v = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad w = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

$$= \begin{pmatrix} \lambda x_1 + \mu y_1 \\ \lambda x_2 + \mu y_2 \end{pmatrix}$$

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Note (usually $v = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ $w = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$

$$\lambda \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + \mu \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} = \begin{pmatrix} \lambda x_1 + \mu x_2 \\ \lambda y_1 + \mu y_2 \end{pmatrix}$$)

<u>Rules</u> 1) $\lambda(v+w) = \lambda v + \lambda w$

2) $v + (-v) = 0$ where $-v$

$v = \begin{pmatrix} x \\ y \end{pmatrix}$ $-v = \begin{pmatrix} -x \\ -y \end{pmatrix}$

3) $v + (w+z)$

$= (v+w) + z$

4) $v + w = w + v$

Here $\lambda \in \mathbb{R}$ $v, w \in \mathbb{R}^2$ (actually $\mathbb{R}^d$)

HW Check that rules 1) to 4) hold

for function spaces

$V = \{ f\ [0,1] \to \mathbb{R} \mid f \text{ continuous} \}$

$\lambda(f+g) = \lambda f + \lambda g$

where $(f+g)(t) = f(t) + g(t)$

and $(\lambda f)(t) = \lambda f(t)$

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The spanning problem in 2D

Problem I Given two vectors v, w in $\mathbb{R}^2$.

When is it true that for all z in $\mathbb{R}^2$ we can find λ, μ such that

$$(*) \quad z = \lambda v + \mu w \quad ?$$

Examples 1) $v = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $w = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

2) $v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $w = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

3) $v = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $w = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

4) $v = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $w = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$

Answer: If v, w do not point in same direction up to sign!

Def $\{v, w\}$ is called a basis for $\mathbb{R}^2$ if every point can be expressed by $(*)$

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