Homework 8

Due date: October 26.

1. Let \( f(x, y) = (e^x y, e^{-x} y) \). Find the center of mass for the image \( f([0, 1]^2) \).

2. Let \( f(x, y) = (3x + y^2, 4x - 3y^2) \). Find the center of mass for the image \( f([0, 1]^2) \).

3. Describe the circle with radius 1/2 and center \((1/2, 0)\) with polar coordinates.

4. Let \( P(x, y) = -y \) and \( Q(x, y) = x \). Calculate
   \[
   \int_{\partial D} P \, dx + Q \, dy
   \]
   in two ways for the Archimedian spiral \( D = \{(r \cos \theta, r \sin(\theta)) : 0 \leq \theta \leq 2\pi, r \leq a\theta\} \).

5. Let \( R = [0, 1]^2 \) and \( P(x, y) = -y^2 \) and \( Q(x, y) = x^2 \). Calculate
   \[
   \int_{\partial R} P \, dx + Q \, dy.
   \]

6. Evaluate
   \[
   \int_0^1 \int_{y^{3/2}}^{1} y^{3/2} \cos(1 + x^2) \, dx \, dy.
   \]