Homework 7

Due Date: Thursday October 20.

1. After class we had a discussion on global maxima and global minima. Let me recall that a domain like

$$D = \{(x,y) : x^2 + y^2 \leq 1\} \quad \text{or} \quad R = [a,b] \times [c,d]$$

is closed (because of the "$\leq$" sign) and bounded.

Thm: Let D be closed and bounded, and $f : D \to \mathbb{R}$ be continuous. Then f attains its maximum and minimum.

Describe a strategy to find all maxima and minima on D or R for a differentiable function! Why do you have to consider two cases?

2. Use the inequalities

$$\underline{\Sigma}(f,P) \leq \int_R f(x,y)dA(x,y) \leq \overline{\Sigma}(f,P).$$

to estimate an integral

i) $f(x,y) = (\cos(x)\sin(y))^{1/3}$ in $R = [0,\pi/2] \times [0,\pi]$ and the partition P given by $t_0 = 0$, $t_1 = \pi/2$, $s_0 = 0$, $s_1 = \pi$.

(a) As before with $t_0 = 0$, $t_1 = \pi/2$, $s_0 = 0$, $s_1 = \pi/2$ and $s_1 = \pi$.

3. Use the definition of integrability to show that for the domain

$$D = \{(x,y) : 0 \leq y \leq x^2, 0 \leq x \leq 1\}$$

the function 1_D is integrable.

4. Given an example of a domain in $\mathbb{R}^2$ which is neither and type I nor type II, and calculate its area.

5. Find the word "graph" in the section on multiple integrals and domains. By the way "graph" comes from greek word "graphein" which means writing or drawing.