Homework 3

Due date: September 15

1. For the following vector functions calculate $\frac{d^2r}{ds^2}$ and curvature. Check that $\frac{d^2r}{ds^2}$ and $\frac{dr}{dt}$ are orthogonal.

   (a) $r(t) = \begin{pmatrix} t \\ t^2 \end{pmatrix}$;

   (b) $r(t) = \begin{pmatrix} e^t \\ e^{2t} \end{pmatrix}$;

   (c) $r(t) = \begin{pmatrix} t^2 \\ t^2 \end{pmatrix}$;

2. Let $r(t) = \begin{pmatrix} t \\ t^2 \end{pmatrix}$.

   (a) Find an orthogonal frame $v, w : [a, b] \to \mathbb{R}^2$ such that v and dr/dt point in the same direction.

   (b) Find a formula for the coordinated of the point $(0, 1)$ as seen from an observer following the vector function and looking in the direction of the velocity vector. Does these coordinates depend on the speed?

   (c) Find a formula for the angle in view an observer looks at $(0, 1)$. Can you do this with less calculation as above?

3. In the book they define the cross product

$$a \times b = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

   for two vectors $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$, $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$. Check a couple of example and explain

$$|a \times b| = \text{area}(P_{a,b})$$

at least for vectors a, b with $a_3 = b_3 = 0$. Here $P_{a,b}$ is the parallelogram spanned by the vectors a and b.

4. If you haven't done so read the chapter in the book about arclength, curvature and moving frame.