Additional assignments

1. Let $a = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$ and $f_a(v) = av$ be the corresponding linear transformation. Find the matrix of f_a with respect to the following basis (spanning systems)

   (a) $v = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $w = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

   (b) $v = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $w = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

   (c) Can you find v and w such that new matrix $\tilde{a} = \begin{pmatrix} * & 0 \\ 0 & * \end{pmatrix}$ is diagonal? (Hint: $v = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is a good start and then you have to try an aritrary w...).

2. Find a path (parametrization) γ for

$$C = \{(x, y) : x^2 + y^2 = 1\}$$

   (a) First with constant speed, aka $|\gamma'(t)| = c$;

   (b) Then with speed $|\gamma'(t)| = e^t$.

3. Let $C > 0$. Find the arclength of $\gamma : [0, 1] \to \mathbb{R}^2$, $\gamma(t) = (t^2, t^3 + C)$. What makes the exponents $(2, 3)$ here special?

4. Review the ε-δ definition of continuity. Find two functions f_1 and f_2 such that f_1 is continuous at 0 and f_2 is not. Prove your claim.