1 Homework and practice problems

1. Calculate the total derivative of the following differential forms (recall that $v_j = dx_j$ and $v_{i_1} \cdots v_{i_k} = dx_{i_1} \wedge \cdots \wedge dx_{i_k}$ are the notation for the same object.

   (a) $x_1 v_2$

   (b) $x_1^2 dx_2 + x_2^2 dx_1$

   (c) $x_1^2 dx_1 \wedge dx_3 + \sqrt{x_2} dx_1 \wedge dx_3 + \sqrt{x_2} dx_1 \wedge dx_2$.

   (d) $(x_1^2 + x_2^2 + x_3^2) dx_1 \wedge dx_2 \wedge dx_3$ in $\mathbb{R}^3$ and in $\mathbb{R}^4$.

2. Let $g \begin{pmatrix} s \\ t \end{pmatrix} = \begin{pmatrix} s \\ t \\ f(s,t) \\ h(s,t) \end{pmatrix}$. Write down a formula which allows you to compute

$$area(g(D))$$

where $D = \{(s,t) | s^2 + t^2 \leq 1\}$ is the standard disc. Calculate the area specifically for $f(s,t) = \int_0^{\sqrt{s^2+t^2}} \sqrt{1 - v^3} dv$ and $h(s,t) = \frac{2}{5}((s^2 + t^2)^{5/4}$.

3. Let

$$g \begin{pmatrix} s \\ t \end{pmatrix} = \begin{pmatrix} s \\ t \\ 1 - s^2 - t^2 \\ 1 - \sqrt{s^2 + t^2} \end{pmatrix},$$

and $S = g(D)$, $D = \{(s,t) : s^2 + t^2 \leq 1\}$ the unit disc

   (a) Let $\phi = x_1 x_2 x_4 dx_3$ Calculate $d\phi$.

   (b) Let $\omega = x_2 x_4 dx_1 \wedge dx_3 + x_1 x_4 dx_2 \wedge dx_3 + x_1 x_2 dx_4 \wedge dx_3$.

   (c) Calculate

$$\int_S d\omega$$

   in three different ways.

4. Find a formula (and evaluate if this looks doable) for the following integrals

(a) $\int_{0\le x_1x_2\le 1, 0\le x_1,x_2\le 1, x_3=\sqrt{x_2}+x_3} x_1x_2dx_1\wedge dx_3$;

(b) $\int_{x_2=x_1^2, x_3=x_1^4, 0\le x_1\le 1} x_1^5dx_3$

(c) $\int_{(x_1x_3)^{x_3}=x_2, 1\le x_1,x_2\le 2} x_3dx_1\wedge dx_s$