

Homework 11

Due date: Thursday November 17.

1. Let $G = \left\{ \begin{pmatrix} uv \\ u + v \\ u + uv \end{pmatrix} : 0 \leq x, y \leq 1 \right\}$ and $V = \begin{pmatrix} 2(z - x) - y - 1 \\ z - x \\ y - 2(z - x) \end{pmatrix}$.

Calculate

$$\int_G V \cdot N \, dS.$$

2. Let $G = \left\{ \begin{pmatrix} uv \\ u + v \\ u + uv \end{pmatrix} : 0 \leq u^2 + (u - v)^2 \leq 1 \right\}$ and $V = \begin{pmatrix} 0 \\ z - x \\ y - 2(z - x) \end{pmatrix}$.

Calculate

$$\int_G \frac{V}{\|V\|} \cdot N \, dS.$$

3. Let $G = g(D)$ be a surface given by a differentiable map

- (a) What do you have to check if you want to know that g is a parametrization.
 (b) Let V be a continuous vector field. Can you change g to \tilde{g} such that $g(D) = \tilde{g}(D)$ and

$$\int_G V \cdot N_g \, dS = - \int_G V \cdot N_{\tilde{g}} \, dS.$$

- (c) In what sense does the vector-valued surface integral depend on parametrization?