Honors Calc-Homework 1

Due date: September 1

i) Multiply the following matrices

i) $a = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, b = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix};$

ii) $a = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}; b = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}.$

ii) Find a 2×2 matrix a such that $a^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \neq a$. The matrix in the middle is called 0 matrix. Also find two matrices a, b such that $ab \neq ba$.

iii) Show that $v_\theta v_\eta = v_{\theta+\eta}$, where

$$v_\theta = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} .$$

iv) For two real numbers a, b we define the matrix

$$m_{(a,b)} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} .$$

Calculate $m_{(a,b)}m_{(c,d)}$ and show that

$$m_{(a,b)}m_{(c,d)} = m_{(c,d)}m_{(a,b)} .$$

Find a, b such that

$$m_{(a,b)}m_{(a,b)} = m_{(-1,0)} .$$

For those who have seen that before, do you see a connection to complex numbers?