i) Multiply the following matrices

i) \[ a = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad b = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}; \]

ii) \[ a = \begin{pmatrix} 1 & 0 \\ 1 & 2 \end{pmatrix}, \quad b = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}. \]

ii) Find a $2 \times 2$ matrix $a$ such that $a^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \neq a$. The matrix in the middle is called 0 matrix. Also find two matrices $a, b$ such that $ab \neq ba$.

iii) Show that $v_\theta v_\eta = v_{\theta+\eta}$, where

\[ v_\theta = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}. \]

iv) For two real numbers $a, b$ we define the matrix

\[ m_{(a,b)} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}. \]

Calculate $m_{(a,b)} m_{(c,d)}$ and show that

\[ m_{(a,b)} m_{(c,d)} = m_{(c,d)} m_{(a,b)}. \]

Find $a, b$ such that

\[ m_{(a,b)} m_{(a,b)} = m_{(-1,0)}. \]

For those who have seen that before, do you see a connection to complex numbers?