

# Graph surfaces

$$g\begin{pmatrix} s \\ t \end{pmatrix} = \begin{pmatrix} s \\ t \\ h(s,t) \end{pmatrix} \quad w = P dx_1 dx_2 + Q dx_2 dx_3 + R dx_3 dx_1$$

$$\det g'_{12} = 1 \quad \begin{pmatrix} \frac{\partial s}{\partial s} & \frac{\partial s}{\partial t} \\ 0 & 1 \end{pmatrix}$$

$$\det g'_{23} = -\frac{\partial h}{\partial s} \quad \det g'_{31} = -\frac{\partial h}{\partial t}$$

$$\int_{g(D)} w = \int_D P(g(s,t)) dA(s,t) + \int_D R(g(s,t)) \left(-\frac{\partial h}{\partial t}\right) dA(s,t) + \int_D Q(g(s,t)) \left(-\frac{\partial h}{\partial s}\right) dA(s,t)$$

Example  $D = \{(s,t) : s^2 + t^2 \leq 1\}$   $h(s,t) = f(\sqrt{s^2 + t^2})$

Then  $(P=0 \quad R=x_2 \quad Q=x_1)$

$$\int_{g(D)} w = \int_D P(g(s,t)) dA(s,t) + \int_D R(g(s,t)) f'(\sqrt{s^2+t^2}) \frac{t}{\sqrt{s^2+t^2}} dA(s,t) - \int_D Q(g(s,t)) f'(\sqrt{s^2+t^2}) \frac{s}{\sqrt{s^2+t^2}} dA(s,t)$$

$$= - \int_D \frac{t^2 + s^2}{\sqrt{s^2+t^2}} f'(\sqrt{s^2+t^2}) dA(s,t) = -2\pi \int_0^1 f'(r) r dr$$