

Theorem a $f_a: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ $f_a(v) = av$

New basis $\{v, w\}$. Then the matrix \tilde{a} of f_a with respect to the new basis is given by

$$\tilde{a} = c^{-1}ac$$

Where $c = (v, w)$ is the unique matrix such that $c\begin{pmatrix} 1 \\ 0 \end{pmatrix} = v$ $c\begin{pmatrix} 0 \\ 1 \end{pmatrix} = w$.

Proof the matrix of \tilde{a}

$$\begin{aligned} f_a(v) &= \lambda v + \mu w \\ f_a(w) &= \gamma v + \delta w \end{aligned} \quad \tilde{a} = \begin{bmatrix} \lambda & \gamma \\ \mu & \delta \end{bmatrix}$$

Recall 1) $f_a(v) = av$ 2) $c^{-1}av = \begin{pmatrix} \lambda \\ \mu \end{pmatrix}$

c

$c^{-1}aw = \begin{pmatrix} \gamma \\ \delta \end{pmatrix}$

$$\begin{aligned} c^{-1}a \begin{pmatrix} v & w \end{pmatrix} &= c^{-1}ac \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} && \text{bc } \begin{cases} c\begin{pmatrix} 1 \\ 0 \end{pmatrix} = v \\ c\begin{pmatrix} 0 \\ 1 \end{pmatrix} = w \end{cases} \\ \parallel & & & \\ \begin{pmatrix} \lambda & \gamma \\ \mu & \delta \end{pmatrix} & & \parallel & \\ c^{-1}ac & & & \square \end{aligned}$$