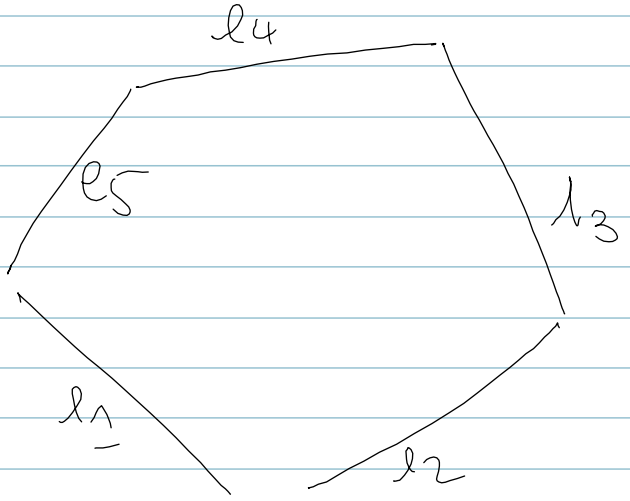
Arc length

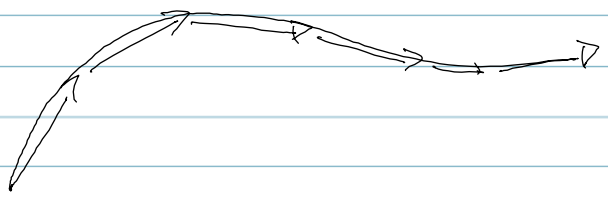
Warning (usually one can not calculate easily)

Theoretically easy



length

$L = l_1 + l_2 + \cdots + l_3$



Length $\approx \sum$ (length of tangent vector) $\cdot \Delta t$

velocity vector

$p(t)$ given $t_0 \le t \le t_1$

$$L(p) = \int_{t_0}^{t_1} \left| \frac{dp}{dt} \right| dt$$

Example $p(t) = (a\cos t + b \sin t)$

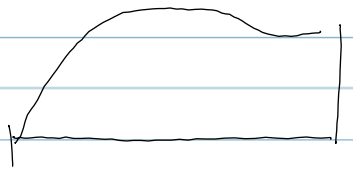
$$p'(t) = (-a \sin t + b \cos t)$$

$$|p'(t)| = \sqrt{a^2 \sin^2 t + b^2 \cos^2 t} \qquad a^2 > b^2$$

$$= \sqrt{a^2 + (b^2 - a^2)\cos^2 t}$$ in general hard

$a = b = 1$ $\int_0^{2\pi} dt = 2\pi$ (at least)

$p(t) = (t, f(t))$



$$\frac{dp}{dt} = \left(1, \frac{df}{dt}\right) \qquad \left|\frac{dp}{dt}\right| = \sqrt{1 + \left(\frac{df}{dt}\right)^2}$$

$$L(p) = \int_{t_0}^{t_1} \sqrt{A + \left(\frac{df}{dx}\right)^2}\, dx \qquad (x = t)$$