1. Let \( f(t) = \frac{1}{2} \), for \( 0 < t < \pi \).

(a) Find the Sine Series of \( f(t) \).

\[
L = \pi \\
b_n = \frac{2}{L} \int_0^L f(t) \sin \frac{n\pi t}{L} \, dt = \frac{2}{\pi} \int_0^\pi \frac{1}{2} \sin nt \, dt \\
= \frac{1}{n\pi} \left[ -\frac{1}{n} \cos nt \right]_0^\pi = -\frac{1}{n\pi} (1 - (-1)^n) \\
= \left\{ \begin{array}{ll}
0 & \text{if } n \text{ even} \\
\frac{2}{n\pi} & \text{if } n \text{ odd}
\end{array} \right.
\]

Sine Series: \( f(t) = \sum_{n \text{ odd}} \frac{2}{n\pi} \sin nt \)

(b) Find the value that the Sine Series of \( f(t) \) converges to at \( t = 1 \).

Since \( f(t) \) is continuous at \( t = 1 \).

By the Convergence Theorem of Fourier Series

the Sine series of \( f(t) \) converges to \( f(1) = \frac{1}{2} \)

(c) Find the Cosine Series of \( f(t) \).

Cosine Series: \( f(t) = \frac{1}{2} \)
2. Let \( f(t) \) be a periodic function with period \( 2\pi \) and \( f = |t| \) for \(-\pi < t < \pi\). Find the Fourier Series of \( f(t) \).

\[
L = \pi
\]

\[
A_0 = \frac{1}{L} \int_{-L}^{L} f(t) \, dt = \frac{1}{\pi} \int_{-\pi}^{\pi} |t| \, dt
\]

\[
= \frac{2}{\pi} \int_{0}^{\pi} t \, dt = \pi
\]

\[
A_n = \frac{1}{L} \int_{-L}^{L} f(t) \cos \frac{n\pi t}{L} \, dt
\]

\[
= \frac{1}{\pi} \int_{-\pi}^{\pi} |t| \cos nt \, dt
\]

\[
= \frac{2}{\pi} \int_{0}^{\pi} t \cos nt \, dt
\]

\[
= \frac{2}{n^2 \pi} \int_{0}^{\pi} u \cos u \, du
\]

\[
= \frac{2}{n^2 \pi} \left( \sin u \bigg|_{0}^{\pi} - \int_{0}^{\pi} \sin u \, du \right)
\]

\[
= \frac{2}{n^2 \pi} \left( 0 - \left[ \frac{-\cos u}{n} \right]_{0}^{\pi} \right)
\]

\[
= \frac{2}{n^2 \pi} \left( 0 - \frac{-1}{n} \right) = \begin{cases} 0 & \text{if } n \text{ even} \\ \frac{2}{n^2 \pi} & \text{if } n \text{ odd} \end{cases}
\]

\[
b_n = \frac{1}{L} \int_{-L}^{L} f(t) \sin \frac{n\pi t}{L} \, dt = \frac{1}{\pi} \int_{-\pi}^{\pi} |t| \sin nt \, dt = 0
\]

**Bonus Question (2 points)**

Find the Fourier Series of \( f(t) \):

\[
\sum_{n \text{ odd}} \frac{1}{n^2} \cos nt
\]

3. Use what you get in Problem 2 to find the value of the following infinite sum:

\[
\frac{\pi^2}{2} + \frac{2}{n \text{ odd}} \frac{-4}{n^2 \pi} = f(0) = 0
\]

\[
\Rightarrow \frac{\pi^2}{2} + \frac{2}{n \text{ odd}} \frac{-4}{n^2 \pi} = f(0) = 0
\]

\[
\Rightarrow \frac{2}{n \text{ odd}} \frac{1}{n^2} = \frac{\pi^2}{8}
\]