

ALGEBRAIC TOPOLOGY I

Igor Mineyev. Math 525, Spring 2019.

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Textbook: *Algebraic topology*, by Allen Hatcher. Freely available online at www.math.cornell.edu/~hatcher/AT/ATpage.html

The tentative syllabus.

Fundamental group and covering spaces.

- (1) Definition of the fundamental group.
- (2) Covering spaces and lifts of maps.
- (3) Computing the fundamental group via covering spaces.
- (4) Applications, such as the Fundamental Theorem of Algebra and the Brouwer fixed point theorem in 2d.
- (5) Deforming spaces: retraction and homotopy equivalence.
- (6) Quotient topology and cell complexes.
- (7) Homotopy extension property and applications to homotopy equivalence.
- (8) Fundamental groups of CW complexes.
- (9) Van Kampen's Theorem.
- (10) Covering spaces and subgroups of the fundamental group.
- (11) Universal covers.
- (12) The definitive lifting criterion, classification of covering spaces.
- (13) Covering transformations and regular covers.

Homology.

- (14) Delta complexes and their cellular homology.
- (15) Singular homology.
- (16) Homotopic maps and homology.
- (17) The long exact sequence of the pair.
- (18) Relative homology and excision.
- (19) Equality of cellular and singular homology.
- (20) Applications, such as degree of maps of spheres, invariance of dimension, and the Brouwer fixed point theorem.
- (21) Homology of CW complexes.
- (22) Homology and the fundamental group: the Hurewicz theorem.
- (23) Euler characteristic.
- (24) Homology with coefficients.
- (25) Intro to categories and axiomatic characterization of homology theories.
- (26) Further applications, such as the Jordan curve theorem, wild spheres, invariance of domain.