

# USTARS

underrepresented students in topology and algebra research symposium

University of California, Berkeley  
Berkeley, California  
April 11-13 2014



# Symposium Agenda

## Friday, April 11

6:00am-10:00pm Arrival to USTARS 2014

## Saturday, April 12

8:00-12:00pm **Registration**  
Durant Hall Atrium

9:00-9:30am **Welcome and Opening Remarks**  
Dr. Candice Price, United States Military Academy, West Point  
Dr. David Eisenbud, Director of MSRI, University of California, Berkeley  
Location: Durant Hall Atrium

9:30-10:00am **Session I**  
*Graded modules over free algebras*  
Guatam Sisodia, University of Washington  
Location: Durant Hall Room 9

*Graph Theoretical Analysis of Knot Distances*  
Annette Honken, University of Iowa  
Location: Durant Hall Room 120

*Example of representation stability phenomena*  
Dr. Rita Jimenez Rolland, Northeastern University  
Location: Durant Hall Atrium

10:15-10:45am **Session II**  
*A Family of Multi-Rees Koszul Cohen-Macaulay Normal Algebras*  
Gabriel Sosa, Purdue University  
Location: Durant Hall Room 9

*Equivariant Homotopy Theory*  
Mychael Sanchez, University of Illinois  
Location: Durant Hall Room 120

*MAPPER as a tool for analyzing data obtained from flow cytometry*  
Leyda Almodovar, University of Iowa  
Location: Durant Hall Atrium

10:45-11:15am **Coffee Break**  
Location: Durant Hall Atrium

11:15-12:30pm **Invited Faculty Speaker**  
*When Algebra Meets Optimization, And My Journey Through Grad School*  
Dr. Mohamed Omar, Harvey Mudd College  
Location: Durant Hall Atrium

12:30-1:30pm **Lunch**  
Location: Durant Hall Atrium

## Saturday, April 12

### 1:30-2:00pm **Session III**

*A generating function for Weyl Dimension Formula*  
Wayne Johnson, University of Wisconsin, Milwaukee  
Location: Durant Hall Room 9

*On the Dubrovinik polynomial of rational knots*  
Katherine Urabe, California State University, Fresno  
Location: Durant Hall Room 120

*Radicals of Extensions*  
Jessica Williams, University of Iowa  
Location: Durant Hall Atrium

### 2:15-2:45pm **Session IV**

*On filtered representations of quivers with at most two pathways and on the generalized Grothendieck-Springer resolution.*  
Mee Seong Im, University of Illinois, Urbana-Champaign  
Location: Durant Hall Room 9

*Classification of tight contact structures using convex surface theory*  
Marcos Ortiz, University of Iowa  
Location: Durant Hall Room 120

*A quantitative lower bound theorem for face numbers of polytopes*  
Jose Alejandro Samper Casas, University of Washington  
Location: Durant Hall Atrium

### 2:45-3:00pm **Coffee Break & Conference Photo**

Location: Durant Hall Atrium

### 3:00-4:00pm **Poster Session**

Location: Durant Hall Atrium

### 4:15-5:30pm **Distinguished Graduate Student**

*Loop groups, embeddings and moduli spaces*  
Pablo Solis, University of California, Berkeley  
Location: Durant Hall Atrium

### 7:00-8:30pm **Symposium Banquet**

Location: Dwinelle Hall Room 370

## Sunday, April 13

9:00-9:30am

### Session V

*Computing Peak Set Probabilities in Coxeter Groups*

Darleen Perez-Lavin, Florida Gulf Coast University

Location: Durant Hall Room 9

*A bound for the number of disjoint genus one surfaces bounded by a hyperbolic knot in the 3-sphere.*

Dr. Luis Valdez-Sanchez, University of Texas at El Paso

Location: Durant Hall Room 120

*Shi Arrangements, Parking Functions and Mixed Graphs*

Ana Berrizbeitia, University of Iowa

Location: Durant Hall Atrium

9:40-10:10am

### Session VI

*Representations of Non-commutative  $W$ -Graphs*

Alexander Diaz-Lopez, University of Notre Dame

Location: Durant Hall Room 9

*Intrinsic Linking in Directed Graphs*

Natalie Rich, University of Nebraska

Location: Durant Hall Room 120

*On  $(t, r)$  Domination Numbers of Grids*

David Blessing, Florida Gulf Coast University

Location: Durant Hall Atrium

10:20-11:30am

### Distinguished Graduate Student

*Galois module structure of abelian extensions*

Cindy (Sin Yi) Tsang, University of California, Santa Barbara

Location: Durant Hall Atrium

11:45am- 1:15pm

### Networking Lunch and Symposium Closing

Location: Dwinelle Hall Room 370

#### Panelists:

Anastasia Chavez, University of California, Berkeley

Dr. Emille D. Lawrence, University of San Francisco

Dr. Stephen Mendez-Diez, University of Alberta

Samuel Ivy, North Carolina State University

Dr. Amanda Ruiz, Harvey Mudd College

**Moderator:** Dr. Syvillia Averett, Central State University

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## Oral Presentation Abstracts

**Title:** MAPPER as a tool for analyzing data obtained from flow cytometry

**Presenter:** Leyda Almodovar

**Affiliation:** University of Iowa

**Abstract:** MAPPER is a computational method for the analysis, simplification and visualization of high dimensional data sets, based on topological ideas created by Gurjeet Singh, Facundo Mémoli and Gunnar Carlsson. It outputs a graph, which captures topological information and can help to visualize specific properties of the data. We used topological methods such as MAPPER as tools for analyzing data collected from flow cytometers.

**Title:** Shi Arrangements, Parking Functions and Mixed Graphs

**Presenter:** Ana Berrizbeitia

**Affiliation:** University of Iowa

**Abstract:** A Shi arrangement is a hyperplane arrangement in  $R^n$  consisting of all planes of the form  $x_i = x_j$  and  $x_i = x_j + 1$  for  $1 \leq i < j \leq n$ . A parking function is an  $n$ -tuple such that once ordered,  $a_i \leq i$ . The two objects seem completely unrelated, but the regions of a Shi arrangement are actually in 1-1 correspondence with the  $n$ -tuples which are parking functions. In this talk, I will motivate the definition of both objects and exhibit an explicit bijection using the aid of mixed graphs. It is a cute and unique solution to an old problem originally solved by Pak and Stanley.

**Title:** On  $(t, r)$  Domination Numbers of Grids

**Presenter:** David Blessing

**Affiliation:** Florida Gulf Coast University

**Abstract:** We find a generalization of domination theory which includes classic domination theory as a special case.  $(t, r)$  Broadcast Domination is such a theory. Each selected vertex in a grid  $G$ ,  $v$ , emits a signal of strength  $t$  to nearby vertices,  $u$ , which decays proportionally to the minimal path connecting  $v$  to  $u$ . A collection of vertices is a  $(t, r)$  Broadcast domination set if all vertices in  $G$  receive a total signal of  $r$  or more. We derive closed form formulas for grids and upper bounds for large grids in  $(2, 2)$ ,  $(3, 1)$ , and  $(3, 2)$  Broadcast Domination.

**Title:** Representations of Non-commutative W-Graphs

**Presenter:** Alexander Diaz-Lopez

**Affiliation:** University of Notre Dame

**Abstract:** In 1979, Kazhdan and Lusztig studied Hecke algebras, which arise as endomorphism algebras of representations of groups induced by representations of subgroups. Later, they define representations of Hecke algebras indexed by left cells of Coxeter groups. In my talk, I will construct representations of Hecke algebras on quotient path algebras over suitable quivers, and discuss their relation with the representations indexed by left cells.

**Title:** Graph Theoretical Analysis of Knot Distances

**Presenter:** Annette Honken

**Affiliation:** University of Iowa

**Abstract:** A knot is the embedding of  $S^1$  in three dimensional space up to ambient isotopy. We define the distance between two knots,  $K$  and  $L$ , to be the minimum number of crossing changes required to obtain  $L$  from a sequence of crossing changes on  $K$ . Then, using rational knots as vertices and placing an edge between two vertices if they are of distance one, we have knot distances graphs. Thus far, we have studied knot distance graphs of rational knots with up to ten crossings. We aim to classify these knots as random, small-world, or scale-free networks through taking measurements of degree, degree distribution, distance statistics, clustering coefficient, and scale-freeness. Additionally, we look at what aligning graphs with different maximum crossing number may tell us. Knots model DNA as well as locally knotted protein, and a crossing change models the action of a type II topoisomerase on double stranded circular DNA. Therefore, we will work to find new patterns and properties among knots via crossing changes and hope to extract biological information as well.

**Title:** On filtered representations of quivers with at most two pathways and on the generalized Grothendieck-Springer resolution

**Presenter:** Mee Seong Im

**Affiliation:** University of Illinois, Urbana-Champaign

**Abstract** Given a group action on a topological space, one studies and classifies the structure of orbits on the space. Using techniques in multilinear algebra, geometric invariant theory, algebraic geometry, homological algebra, and symplectic geometry, representations of quivers with the natural group action generalize classical results, such as Procesi and Donkin's study of Lie group actions on Lie algebras and the Kronecker problem. A world of interesting problems arises, leading us to Nakajima's quiver varieties, Gan and Ginzburg's almost-commuting variety in representation theory, optimization and control in compactification of moduli of linear dynamical systems, etc. I will discuss the history of equivariant geometry, introduce the notion of filtered quiver varieties, and explain their relation to an important object in representation theory called the Grothendieck-Springer resolution. New definitions will be introduced which will give us an algebraic condition that if  $\mathbb{U}$ -invariants for filtered representations of nonframed quivers only come from diagonal blocks of representations (i.e., the semisimple part), then  $Q$  has at most two pathways between any two vertices. I will then discuss how to obtain the Hamiltonian reduction of the cotangent bundle of the Grothendieck-Springer resolution from a filtered quiver variety and I will give some results. I will conclude with some open problems and conjectures relating to filtered representations of quivers and on the generalized Grothendieck-Springer resolution.

**Title:** Examples of representation stability phenomena

**Presenter:** Dr. Rita Jimenez Rolland

**Affiliation:** Northeastern University

**Abstract:** We will focus on examples of sequences of spaces or groups  $X_n$  whose rational cohomology carries an action of the symmetric group  $S_n$ . We will see that these  $S_n$ -representations "stabilize" when the parameter  $n$  is sufficiently large.

**Title:** A generating function for the Weyl Dimension Formula

**Presenter:** Wayne Johnson

**Affiliation:** University of Wisconsin, Milwaukee

**Abstract:** In 2011, Gross and Wallach published a closed form for the Hilbert series of an equivariantly embedded projective variety. In this talk, we present a generalization of their result that gives a generating function for the dimensions of the irreducible representations of a semi-simple, simply connected complex linear algebraic group whose highest weight is in any lattice cone in the dominant chamber.

**Title:** When Algebra Meets Optimization, And My Journey Through Grad School

**Presenter:** Dr. Mohamed Omar \*\*

**Affiliation:** Harvey Mudd College

**Abstract:** In recent years, algebraic tools have played a fundamental role in understanding theory and algorithms in optimization. In this talk, we showcase some particular examples where the interplay between these areas comes to light. We focus on results that came to fruition during the speaker's graduate school experience.

**Title:** Classification of tight contact structures using convex surface theory

**Presenter:** Marcos Ortiz

**Affiliation:** University of Iowa

**Abstract:** Every 3-manifold supports a contact structure but the classification of such structures is largely incomplete outside of a relatively small collection of manifolds. For example, little is known about contact structures on thickened surfaces. One strategy we can employ towards the goal of classifying these structures is to prescribe a dividing set on the boundary of such a manifold and use convex surface theory techniques to find bounds the number of possibly distinct contact structures that give rise to the chosen dividing set. If a reasonably small bound can be found, one can try to explicitly construct the contact structures supported by those manifolds, completing the classification in these cases. In this talk I will describe several of the techniques employed in finding a bound on the number of contact structures. I will summarize some known results and include some original results from my research.

**Title:** Computing Peak Set Probabilities in Coxeter Groups

**Presenter:** Darleen Perez-Lavin

**Affiliation:** Florida Gulf Coast University

**Abstract:** In this talk we explore the question: What is the probability that a random element of a Coxeter group has a given peak set? This question was first explored in type A Coxeter groups by Billey, Burdzy, Pal, and Sagan in 2013. We will discuss results in other Lie types.

**Title:** Intrinsic Linking in Directed Graphs

**Presenter:** Natalie Rich

**Affiliation:** University of Nebraska

**Abstract:** We extend the notion of intrinsic linking to directed graphs. We give methods of constructing intrinsically linked directed graphs, as well as directed graphs with arbitrarily large numbers of edges that are not intrinsically linked. We give a simple construction that takes a graph  $G$  and doubles it to get a directed graph  $\Gamma$  with twice as many edges as  $G$ , proving that it gives an intrinsically linked directed graph if and only if  $G$  is intrinsically linked. One corollary is that  $J_6$ , the complete directed graph on six vertices (with 30 directed edges), is intrinsically linked.

**Title:** A quantitative lower bound theorem for face numbers of polytopes

**Presenter:** Jose Alejandro Samper Casas

**Affiliation:** University of Washington

**Abstract:** The face numbers of simplicial polytopes that approximate  $C^1$ -convex bodies in the Hausdorff metric is studied. We use several techniques from differential/algebraic topology to prove structural results about the skeleta of such polytopes are studied and used to derive a lower bound theorem for this class of polytopes. This partially resolves a conjecture made by Kalai in 1994: if a sequence  $\{P_n\}_{n=0}^\infty$  of simplicial polytopes converges to a  $C^1$ -convex body in the Hausdorff distance, then the entries of the  $g$ -vector of  $P_n$  converge to infinity.

**Title:** Equivariant homotopy theory

**Presenter:** Mychael Sanchez

**Affiliation:** University of Illinois

**Abstract:** This will be a general talk on equivariant homotopy theory with the goal of introducing equivariant K-theory.

**Title:** Graded modules over free algebras

**Presenter:** Gautam Sisodia

**Affiliation:** University of Washington

**Abstract:** Let  $F$  be a finitely generated  $N$ -graded free algebra over a field. The Hilbert series of a graded vector space  $V$  is a formal power series that encodes the dimensions of the homogeneous components of  $V$ . We give an answer to the following question: what are the possible Hilbert series of finitely presented graded  $F$ -modules? We use our answer to compute the Grothendieck group (as an ordered abelian group) of the quotient category of finitely presented graded  $F$ -modules modulo those that are finite dimensional. This work is joint with S. Paul Smith.

**Title:** Loop groups, embeddings and moduli spaces

**Presenter:** Pablo Solis\*

**Affiliation:** University of California, Berkeley

**Abstract:** This talk is about three things: infinite dimensional Lie groups, representation theory and moduli problems. The group in question is called the loop group; it is the space of maps from a circle into a topological group. The loop group has a class of representations -those of positive energy- that generalize the highest weight representations of semisimple groups over the complex numbers. These representations allow one to construct an infinite dimensional space that one can relate to the finite dimensional moduli problem of parametrizing vector bundles on a compact Riemann surface. I'll explain this setup and show how it can be used to compactify this moduli space of vector bundles when the Riemann surface is allowed to develop singularities. Over the last 30 years other compactifications have been presented by Caparoso, Gieseker, Pandharipande, Seshadri and Nagaraj of varying generality but this is the first construction that provides a compactification not just for vector bundles but



for principal  $G$  bundles for an arbitrary simple group over the complex numbers.

**Title:** A Family of Multi-Rees Koszul Cohen-Macaulay Normal

**Presenter:** Gabriel Sosa

**Affiliation:** Purdue University

**Abstract:** Generalizing techniques that prove that Veronese subrings are Koszul, we show that Rees and multi-Rees algebras corresponding to sets that are closed under comparability are Koszul. We provide Grobner basis for the defining ideals of these multi-Rees algebras with squarefree initial monomials, to show that they are also normal Cohen-Macaulay domains. The talk will be based on results from a paper that I will submit to the arxiv later this month (January)

**Title:** Galois module structure of abelian extensions

**Presenter:** Cindy (Sin Yi) Tsang\*

**Affiliation:** University of California, Santa Barbara

**Abstract:** Let  $K$  be a number field with ring of integers  $\mathcal{O}$  and  $G$  a finite group. If  $L/K$  is a Galois extension with group  $G$ , then  $L$  has a  $K$ -basis of the form  $\{\sigma a \mid \sigma \in G\}$ , called a normal basis. Equivalently, this means that  $L$  is free as a  $KG$ -module. A natural question to ask is whether one can generalize this to the ring of integers, i.e. is  $\mathcal{O}_L$  free over  $\mathcal{O}G$ ? It turns out that the global and local freeness of  $\mathcal{O}_L$  is closely related to the ramification of  $L/K$ . Furthermore, when  $\mathcal{O}_L$  is locally free over  $\mathcal{O}G$ , it defines a class in the locally free class group  $\text{Cl}(\mathcal{O}G)$  of  $\mathcal{O}G$ . A natural question to ask is whether the collection of all these classes form a group. In this talk, I will discuss results that have been known to these two questions; I will also briefly explain how one can generalize all of these to the square root of the inverse different.

**Title:** On the Dubrovnik polynomial of rational knots

**Presenter:** Katherine Urabe

**Affiliation:** California State University, Fresno

**Abstract:** Rational knots and links arise as numerator closures of rational tangles, which form a basis for their classification. A rational tangle is the result of consecutive twists on neighboring endpoints of two trivial arcs, and is associated in a canonical manner with a unique, reduced rational number or infinity. This number equals a continued fraction with all numerators equal to one and all denominators of the same sign, that can be read from a particular diagram of the tangle. Equivalently, one can associate to a rational tangle diagram (in standard form) a vector of integers. In this work, we find a closed formula for the Dubrovnik polynomial of a rational knot/link diagram in terms of the entries of its associated vector.

**Title:** A bound for the number of disjoint genus one surfaces bounded by a hyperbolic knot in the 3-sphere.

**Presenter:** Dr. Luis Valdez-Sanchez

**Affiliation:** University of Texas at El Paso

**Abstract:** A genus one knot in the 3-sphere is a knot which bounds a once-punctured torus in its exterior but does not bound a disk. It is known that a hyperbolic knot can have at most 7 disjoint such surfaces in its exterior, and in this talk we will show that in fact at most 6 such surfaces may exist. The proof makes use of combinatorial arguments applied to the graphs obtained by the transverse intersection of suitable surfaces properly embedded in the exterior of a hyperbolic knot, and of the connections between the algebra and the topology of genus 2 handlebodies. Examples of genus one hyperbolic knots whose exterior contain up to 5 disjoint once-punctured tori will also be presented.

**Title:** Radicals of Extensions

**Presenter:** Jessica Williams

**Affiliation:** University of Iowa

**Abstract:** The Jacobson radical of an extension of one module by another is something about which little appears to be known. Victor Camillo, Miodrag Iovanov, and I have been working towards finding properties of this radical. We have been able to prove some injectivity properties under various different conditions for the modules in the sequence and the underlying ring structure. Specifically, we have results for principal ideal domains and Dedekind domains.

## Poster Presentation Abstracts

**Title:** Invariants for Spatial Graphs

**Presenter:** Elaina Aceves and Jennifer Elder

**Affiliation:** California State University, Fresno

**Abstract:** A spatial graph is an embedding of a graph in three-dimensional space. We construct an invariant for spatial graphs by performing certain replacements at the vertices of a graph diagram, which results in a collection  $C$  of arcs and knot/link diagrams. After discarding the arcs, we use known polynomial invariants for knots and links to evaluate the objects in the collection  $C$ , and obtain a Laurent polynomial associated with our original spatial graph, which is independent on the embedding type of the graph. Thus our approach yields an invariant for spatial graphs. We discuss some properties of this invariant, including a relationship between the resulting invariant of a spatial graph  $G$  and the invariants associated with the two graphs obtained from  $G$  by applying the contraction-deletion move in a neighborhood of a vertex of the graph  $G$ .

**Title:** Permutation Patterns for Real-Valued Functions

**Presenters:** Alicia Arrua, Gustavo Meléndez Ríos, and Lynesia Taylor

**Affiliation:** California State Polytechnic University, Pomona; University of Puerto Rico, Rio Piedras; Spelman College

**Abstract:** Let  $[x, f(x), f(f(x)) = f^2(x), \dots, f^{(n-1)}(x)]$  be the sequence where  $f$  is a real-valued function and  $n \geq 2$ . We can associate a permutation to every such sequence by comparing it with  $x_1 < x_2 < \dots < x_n$ , where  $x_i = f^{(j-1)}(x)$  for some  $j = 1, 2, \dots, n$ . Permutations that occur are allowed permutations; otherwise they are called forbidden permutations. We focus on enumerating and identifying permutations of real-valued functions such as the logistic map ( $L_r(x) = rx(1-x)$  with  $0 < r \leq 4$ ), the half-circle map ( $H(x) = \sqrt{x-x^2}$ ), the sine map ( $S(x) = \sin(\pi x)$ ), the castle map ( $C(x) = -16x^4 + 32x^3 - 24x^2 + 8x$ ), and variations of the logistic map ( $L_r(x^k) = rx^k(1-x^k)$  where  $k \geq 2$ ). In the case of the half-circle map, we completely enumerated all of its permutations. Furthermore, we have conjectured that the sine and castle maps are order isomorphic to the  $L_4(x)$ , i.e., they have the same allowed and forbidden permutations. We have also conjectured that several variations of the logistic map are order isomorphic to each other.

**Title:** Computations In The Relative Skein Algebra of a Local Annulus

**Presenter:** Nelson Colon

**Affiliation:** University of Iowa

**Abstract:** Let  $M$  be a compact, oriented, 3-manifold of the form  $F \times [0, 1]$  where  $F$  is an oriented surface of genus  $g$  and  $b$  boundary components.  $K_A(M)$  is the skein algebra of  $M$  for  $A$  an  $N$ -th root of unity. The purpose of this poster is to present tools to reduced a skein in  $K_A(M)$  by finding an annulus in  $F$  and simplifying the skein locally in such annulus.

**Title:** The Algebra of Block Permutations

**Presenters:** Ryan Contreras and Isabel Corona

**Affiliation:** Columbia University; Metropolitan State University of Denver

**Abstract:** A *block permutation* of  $[n] = \{1, 2, \dots, n\}$  consists of two set partitions  $\mathcal{A}, \mathcal{B} \vdash [n]$  having the same number of blocks and a bijection  $f : \mathcal{A} \rightarrow \mathcal{B}$ . We show that the set of block permutations of  $[n]$ ,  $BP_n$ , is closed under multiplication and hence is a monoid. We define a Hopf algebra of block permutations, which is a generalization of the Hopf algebra of uniform block permutations. The algebra  $BP_n$  is a subalgebra of the partition algebra and is therefore a diagram algebra. Furthermore, we study the subalgebra of planar diagrams,  $P_n$ , and give a presentation for both  $P_n$  and  $BP_n$ . We consider the planar rook algebra  $RP_n$  and show that it is isomorphic to  $P_{n+1}$ . Using this isomorphism we can give all the irreducible representations for  $P_{n+1}$ . In addition, we are in the process of constructing the irreducible representations for  $BP_n$ , which will be indexed by Young diagrams with  $1, 2, \dots, n$  boxes. We conjecture as to the degree of the irreducible representations and investigate the induction of representations of  $BP_n$  to  $BP_{n+1}$ .

**Title:** On the Schur Positivity of Differences of Products of Schur Functions

**Presenter:** Jeremiah Emidih and Nadine Jansen

**Affiliation:** University of California, Riverside; North Carolina A&T State University

**Abstract:** The Schur functions are a basis for the ring of symmetric functions indexed by partitions of nonnegative integers. A symmetric function  $f$  is called Schur positive if when expressed as a linear combination of Schur functions

$$f = \sum_{\lambda} c_{\lambda} s_{\lambda}$$

each coefficient  $c_{\lambda}$  is nonnegative. We wish to investigate expressions of the form

$$s_{\lambda} s_{\lambda^c} - s_{\mu} s_{\mu^c}$$

where  $\lambda$  partitions  $n$  and  $\mu$  partitions  $n - 1$  and the complements  $\lambda^c, \mu^c$  are taken over a sufficiently large  $m \times m$  square. We give a necessary condition that if  $s_{\lambda} s_{\lambda^c} - s_{\mu} s_{\mu^c}$  is Schur positive, then  $\mu$  is contained in  $\lambda$ . Furthermore, we show how both increasing the size of our box and conjugating partitions preserves Schur positivity. Lastly, we incorporate the Littlewood Richardson rule to conjecture a full characterization of when  $s_{\lambda} s_{\lambda^c} - s_{\mu} s_{\mu^c}$  is Schur positive.

**Title:** Understand Ultrafilters, Understand Schur's Theorem

**Presenter:** Joseph Foss

**Affiliation:** United States Military Academy, West Point

**Abstract:** Looking at a finite partition of the natural numbers, Schur's Theorem states there exists a cell of the partition such that  $x, y$ , and  $z$  are all members of said cell where  $z = x + y$ . This is an important theoretical result of Ramsey Theory and combinatorics in general. A filter is a set of sets that is closed under finite intersections, closed under supersets, and does not have the empty set as one of its members. An ultrafilter is a maximum filter. By studying semigroups, namely the set of ultrafilters on the natural numbers, we can produce an algebraic proof of Schur's Theorem.

**Title:** Invariants for Virtual Singular Links

**Presenter:** Kelsey Friesen and Thoa Tran

**Affiliation:** California State University, Fresno

**Abstract:** A singular link is an immersion of a disjoint union of circles into three-dimensional space, which admits only finitely many singularities that are all transverse double points. A singular link diagram is a projection of a singular link into a plane, and contains two types of crossings, namely classical crossings and singular crossings.

Virtual knot theory, introduced by Lou Kauffman in 1996, can be regarded as a projection of classical knot theory in thickened surfaces. We take one step further by studying virtual singular links, which can be thought as immersions of disjoint unions of circles into thickened surfaces. A virtual singular link diagram contains then three types of crossings: classical, singular, and virtual crossings.

We focus on constructing invariants for virtual singular links, by means of extending known invariants for classical knots and links.

**Title:** (4,1)- Broadcast Domination of an  $m \times n$  Grid

**Presenter:** Armando Grez

**Affiliation:** Florida Gulf Coast University

**Abstract:** In graph theory a dominating set is a subset of vertices  $S$  such that all vertices in the graph is either in  $S$  or adjacent to some vertex in  $S$ . The domination number is the cardinality of the smallest dominating set of  $G$ . This study examines grid graphs given a broadcast center has a transmission of weight 4 and we wish for all the vertices in the graph have a weight no less than 1 or be covered by more than a single center. We go further to discover an upper bound for reducing (4,1) dominating sets by a determined construction from a perfectly dominated grid in  $Z \times Z$ . In addition, from this construction we derive several other properties for (4,1) graph grids. We also discuss consequences of this construction for (4,1) in finding smaller dominating sets to better approximate the domination number for large grids.

**Title:** Finitary rings, Modules, and orbits of the regular action

**Presenter:** Gerard Koffi

**Affiliation:** University of Iowa

**Abstract:** Given a ring  $R$  with 1, let  $M$  be a left  $R$ -module. We denote the group of units of  $R$  by  $G$ . Then  $G$  acts naturally on  $M$  as follows:  $g * m = gm$  for all  $g$  in  $G$  and  $m$  in  $M$ . This action is called the left regular action. When  $M$  is the left regular module  $R$ , Y. Hirano showed that a ring with finitely many orbits under the regular action is a direct sum of finitely many uniserial rings and a finite ring. The purpose of this paper is to examine the situation when  $M$  is a left  $R$ -module with finitely many orbits under the left regular action. Specifically, the problem is to describe the left  $R$ -modules with finitely many orbits under the left regular action. We prove that when  $R$  is a semilocal ring, such modules have finite length and cannot have a subfactor isomorphic to  $T + T$  where  $T$  is simple and  $\text{End}_R(T)$  is infinite. We use this Theorem to give a module theoretical proof of Hirano's result which was proved using ring theoretical approach.

**Title:** Redefining Patrol Areas: Minimizing Numbers, Maximizing Results

**Presenter:** Victoria Markow

**Affiliation:** United States Military Academy, Westpoint

**Abstract:** During my summer training in 2013, I shadowed a platoon leader at Fort Bragg, North Carolina for three weeks. During this short time period, I witnessed several Military Police platoon leaders struggle to meet the minimum requirements for the number of soldiers patrolling the roads. As one may imagine this problem is not unique to this specific company or even to us, the Army. In fact, as the national budget shrinks, so do the budgets of all the different law enforcement entities. After speaking to my hometown police chief in New Britain, Connecticut, I learned that currently in order to achieve an effective amount of patrols on the roads, superiors are relying on experience and guess work to develop patrol schedules. Although this strategy may be effective, it may not be the most efficient. Due to continuing budgetary constraints, now more than ever it is critical to develop mathematical methods to ensure that police patrols are set in the most optimal way.

**Title:** (2,2)-Broadcast Domination Number of Directed and Undirected grids

**Presenter:** Christie Mauretour

**Affiliation:** Florida Gulf Coast University

**Abstract:** In this presentation, we explore the double domination numbers grids. For an  $m \times n$  grid  $G_{m,n}$ , suppose that a police officer can protect the vertex she is stationed at with weight 2 and every neighboring vertex with weight one. The double domination number of  $G_{m,n}$  is the minimum number of police officers needed to protect the grid with a weight of at least 2 at each vertex. We present closed formulas for the double domination number of grids  $G_{m,n}$  when  $m = 3, 4, 5$ , and  $n \geq m$ . We also present closed formulas for the double domination number of directed  $mn$  grids that model one-way street grids.

**Title:** The Colorability of Crease Patterns

**Presenter:** Esther Oh

**Affiliation:** United States Military Academy, West Point

**Abstract:** We explore graph theory to explain geometric structures of origami and color patterns. Our goal is to demonstrate the colorability of one, two, and three-dimensional origami. First, we examine vertex-degree and how it affects our ability to color each vertex such that no two vertices connected by an edge are colored by the same color. Then, we color crease patterns so that no adjacent regions have the same color. Lastly, by folding three-dimensional origami, we explore how edge-coloring is related to Hamiltonian circuits. Through showing the colorability of origami structures, this research highlights the relationship between the mathematics and folding paper.

**Title:** Number of Permutations with Same Peak Set for Signed Permutations

**Presenter:** Jose Pastrana and Rita Zevallos

**Affiliation:** University of Puerto Rico, Rio Piedras; Swarthmore College

**Abstract:** A signed permutation in the hyperoctahedral group  $B_n$  is a sequence  $\pi_1\pi_2\dots\pi_n$  such that each  $\pi_i \in \{-n, \dots, -1, 1, \dots, n\}$  and  $\{|\pi_1|, |\pi_2|, \dots, |\pi_n|\} = [n] = \{1, 2, \dots, n\}$ . A signed permutation has a peak in a position  $i = 2, \dots, n-1$  if  $\pi_{i-1} < \pi_i > \pi_{i+1}$ . Let  $P(\pi)$  be the set of peaks of  $\pi$ ,  $P_B(S, n)$  be the set of signed permutations  $\pi \in B_n$  such that  $P(\pi) = S$ , and  $\#P_B(S, n)$  be the cardinality of  $P_B(S, n)$ . In [?], Billey, et. al. find

a method for computing the number of permutations with a given peak set  $S$  when working in  $S_n$ . We extend these results to  $B_n$  by showing  $\#P_B(S, n) = p(n)2^{2n-|S|-1}$  where  $p(n)$  is the same polynomial as that given by Billey et. al., and find polynomials for various special cases of  $S$ . In addition, we extend these results both in  $B_n$  and  $S_n$  to the assumption that  $\pi_0 = 0$ , which allows the possibility of a peak at 1.

**Title:** Chromatic Symmetric Functions and Graph Gluing

**Presenter:** Caprice Stanley

**Affiliation:** The George Washington University

**Abstract:** Graph coloring has been and continues to be an active area of study. For a graph  $G$  there is an associated chromatic polynomial,  $\chi(G, x)$ , which gives information about the number of proper colorings for  $G$  if we limit the number of labels to be used. In 1995, Richard Stanley introduced a generalization of the chromatic polynomial. For a graph  $G$ , he defined the chromatic symmetric function as follows,

$$X_G = X_G(x_1, x_2, \dots) = \sum_{\kappa} \left( \prod_{v \in V} x_{\kappa(v)} \right)$$

where the sum is over all proper colorings  $\kappa$  and  $V$  is the set of vertices in  $G$ . From  $X_G$ , there is some information that can be obtained about  $G$  that is not available from  $\chi(G, x)$ . Further, it is possible for two non-isomorphic graphs to share a chromatic polynomial but have different chromatic symmetric functions. It has been a goal to provide sufficient conditions under which two graphs will have the same chromatic symmetric function. We will present some ways to construct graphs that have the same chromatic symmetric functions using the operation of gluing two graphs together at a vertex.