(24) Sketch the graph and find corresponding $x$-$y$ equation.

$$y = 2 \cos \theta.$$

**Sol.**

$$y = 2 \cos x.$$

(28) Sketch the graph.

Find $\theta$ where $r = 0$

Find the range of $\theta$ that covers the graph once.

$$r = 3 - 6 \cos \theta.$$

**Sol.**

$$y = 3 - 6 \cos x.$$
62. Find a polar equation.

\[ x = 2. \]

Sol) \[ x = r \cos \theta. \]

\[ r \cos \theta = 2 \]

So \[ r = \frac{2}{\cos \theta}. \]

\[ \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2} + \frac{\pi}{12} = \frac{7\pi}{12} \]

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\[ \frac{\pi}{4} \]

Find the slope of the tangent line at the given point.

\[ r = 3 \sin \theta \] at \[ \theta = \frac{\pi}{2} \]

Sol) \[ x = r \cos \theta = 3 \sin \theta \cos \theta \]

\[ y = r \sin \theta = 3 \sin^2 \theta. \]

\[ \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{6 \sin \theta \cos \theta}{3 \cos^2 \theta - 3 \sin^2 \theta}. \]

\[ \frac{dy}{dx} \bigg|_{\theta = \frac{\pi}{2}} = 0 \]

So slope = 0.

\[ \frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} + \frac{\pi}{12} = \frac{7\pi}{12} \]

2) Find the area of the indicated region.

Large loop of \[ r = 1 + 2 \sin (2\theta). \]

Sol) \[ y = 1 + 2 \sin (2\theta) \]

3) Find the arc length.

\[ r = 2 \cos (3\theta). \]
Sol) \( y = 2 \cos(3x) \)

\[ \frac{2}{3} E\left(3 \theta \mid -8\right) \]

Elliptic integral of the second kind.

So \( 0 \leq \theta \leq \pi \) cover the graph (three leaves) once.

arc length = six times the first piece of curve which is covered by the angles \( 0 \leq \theta \leq \frac{\pi}{6} \).

So

\[ S = 6 \int_{0}^{\frac{\pi}{6}} \sqrt{\left[\frac{d}{d\theta}(x(\theta))\right]^2 + \left[\frac{d}{d\theta}(y(\theta))\right]^2} \, d\theta. \]

\[ = 6 \int_{0}^{\frac{\pi}{6}} \sqrt{\left[-6 \sin(3\theta)\right]^2 + [2 \cos(3\theta)]^2} \, d\theta. \]

\[ = 6 \int_{0}^{\frac{\pi}{6}} \sqrt{36 \sin^2(3\theta) + 4 \cos^2(3\theta)} \, d\theta. \]

\[ = 6 \int_{0}^{\frac{\pi}{6}} \sqrt{4 + 32 \sin^2(3\theta)} \, d\theta. \]

\( \text{<CAS> } \)

\[ \approx \]