Solution of 1.3.9.

S and T are denumerable, so SUT won't be finite. Therefore, to prove SUT is denumerable, we only need to show SUT is countable.

Since S and T are denumerable, we have bijections f and g such that \( f : \mathbb{N} \to S \), \( g : \mathbb{N} \to T \) are bijections.

Define \( h : \mathbb{N} \to SUT \) as follows.

\[ h(2n+1) = f(n) \quad \text{for any } n \in \mathbb{N} \]
\[ h(2m) = g(m) \quad \text{for any } m \in \mathbb{N}. \]

Then it's easy to see that \( h \) is a surjection from \( \mathbb{N} \) onto \( SUT \). By Theorem 1.3.10, SUT is a countable set. Since it's infinite, SUT is denumerable.

Here is why \( h \) is a surjection. For any \( x \in SUT \),

if \( x \in S \) then \( \exists n \in \mathbb{N} \) s.t. \( f(n) = x \)

so \( \exists 2n+1 \in \mathbb{N} \) s.t. \( h(2n+1) = x \)

if \( x \in T \) then \( \exists m \in \mathbb{N} \) s.t. \( g(m) = x \)

so we find \( 2m \in \mathbb{N} \) s.t. \( h(2m) = x \)

Therefore \( h \) is a surjection.