1. (14pts)
   Find the following limits: (no procedure needed)
   (a) \( \lim_{x \to 2} \frac{x}{1+x} - x^2. \)

   (b) \( \lim_{x \to 1} \frac{\sqrt{1+8x} - \sqrt{1+3x}}{1+2x^2}. \)

   (c) \( \lim_{x \to 0} x \sin \left( \frac{1}{x} \right). \)

   Find the derivative of the following function: (no procedure needed)
   (d) \( f(x) = \frac{x}{1-x}, x \neq \pm1. \)

   (e) \( f(x) = (\sin x^2)^3. \)

   (f) \( f(x) = \tan x^3, -1 < x < 1. \)

   (g) \( f(x) = \arccos x, -1 < x < 1. \)
2. (18 pts) Consider $f(x) = 2|x|$, $x \in \mathbb{R}$

(i) Use definition to prove that $f$ is continuous on $\mathbb{R}$.

(ii) Use definition to find the derivative of $f(x)$ for $x \neq 0$. 
(iii) Prove that $f$ is not differentiable at $x = 0$.

3. (18 pts) Let $f(x) = \sin\left(\frac{1}{x}\right)$ for $x > 0$.

(i) (3pts) We know $\sin x$, $x$ is continuous for $x \in \mathbb{R}$. Is $f(x)$ continuous on $(0, \infty)$? Why?

(ii) (3pts) Given $\sin\left(\frac{\pi}{4}\right) = \sin\left(\frac{3\pi}{4}\right) = 0.707$, $\sin\frac{\pi}{2} = 1$, prove that there exist at least two $x$, $\frac{4}{3\pi} < x < \frac{4}{3\pi}$ such that $f(x) = 0.8$ without using arcsin function.
(iii) (6pts) Prove that $f(x)$ is not uniformly continuous on $(0, \frac{1}{\pi})$.

(iv) (3pts) Let $g$ be a continuous function defined on $[-1, 1]$, $g(x) > 0$. Is the composition $g \circ f$ uniformly continuous on $[\frac{1}{\pi}, \frac{2}{\pi}]$? Why?

(v) (3pts) Prove that there exist a positive number $\alpha$ such that $g \circ f(x) > \alpha$ for all $x > 0$. 
Bonus questions:

(i) (2pts) Show that if $f$ is continuous on $(0, 1)$ then $f^2$ is uniformly continuous on $(0, 1)$ by using the properties of continuous functions from Chapter 5.

(ii) (2pts) Use the knowledge we learned recently to prove that $\sin^2 x + \cos^2 x = 1$ for all $x \in \mathbb{R}$ provided $\sin^2 0 + \cos^2 0 = 1$. 