1. **Homework due Tuesday, August 29, 9 a.m.**

   - Section 1.1: #12, 14, 26, 28, and 36.
   - Section 1.2: #4, 6, 10, 20, and 26.

2. **Homework due Thursday, August 31 at 9 a.m.**

   - Section 1.3: #4, 8, 10, 12, 14, and 16.
   - Section 1.4: #14, 16, 22, and 24.

3. **Notation**

   I will often write “iff” as an abbreviation of “if and only if”, that is, $\iff$.

4. **Absolute Value**

   An important example of a function is the **absolute value** function. Recall that $|a| = a$ if $a \geq 0$ and $|a| = -a$ if $a < 0$. This means that $|a|$ is the distance of $a$ from 0. We also have $|a| = |-a|$, and $|a| = \sqrt{a^2}$.

   Given an inequality involving a variable such as $x$, you often want to know what points on the real line $\mathbb{R}$ satisfy the inequality, that is, make it true. Often the inequality will involve an absolute value. Now, $|a| \leq c$ iff $-c \leq a \leq c$. So $|x - b| \leq c$ iff $-c \leq x - b \leq c$, or $b - c \leq x \leq b + c$. This means that $x$ can get at most a distance $c$ from $b$.

   The **triangle inequality** says $|a + b| \leq |a| + |b|$. We have $|a + b| = |a| + |b|$ if and only if $a$ and $b$ have the same sign. The inequality is named “the triangle inequality” because the same inequality holds for the length of line segments forming a triangle using the appropriate definition of addition.

   An immediate corollary of the triangle inequality is

   $$||a| - |b|| \leq |a - b|.$$  

   This follows since $|a| = |a - b + b| \leq |a - b| + |b|$, so $|a| - |b| \leq |a - b| = |-1(a - b)| = |b - a|$, and the same is true with the roles of $a$ and $b$ reversed.

5. **Cartesian plane**

   I will assume that you are familiar with the Cartesian coordinate system. You should know and understand the Distance Formula:

   $$\text{The distance between } (x_1, y_1) \text{ and } (x_2, y_2) \text{ is } \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$  

   Also you should know and understand the Midpoint Formula for the point $(\bar{x}, \bar{y})$ on the line segment joining $(x_1, y_1)$ and $(x_2, y_2)$ that is half way between these points:

   $$\bar{x} = \frac{x_1 + x_2}{2}, \quad \bar{y} = \frac{y_1 + y_2}{2}.$$
6. Lines in the plane

A line in the Cartesian plane is a geometric object. We can then find, or try to find, a function for which it is the graph. Such a function is called a linear function. A vertical line will not give \( y \) as a function of \( x \). For lines that are not vertical, we define the slope \( m \) by finding any pair of distinct points \((x_1, y_1)\) and \((x_2, y_2)\) on the line and setting

\[
m = \frac{y_2 - y_1}{x_2 - x_1}.
\]

We sometimes write

\[
m = \frac{\Delta y}{\Delta x}
\]

where \( \Delta y \) denotes the change in \( y \) and \( \Delta x \) denotes the change in \( x \). In using the formula \( m = \frac{\Delta y}{\Delta x} \), be careful to keep the order of the \( x \)’s the same as the order of the \( y \)’s.

**NOTE:** The slope of a non-vertical line is the tangent of the angle the line makes with the \( x \)-axis.

Using similar triangles, it is easy to see that the choice of points does not matter in finding the slope.

**EXAMPLES:** If a line goes through the points \((-2, 1)\) and \((3, -5)\), then the slope of the line is \((-5 - 1)/(3 - (-2)) = -6/5\). A horizontal line has slope 0 since for any pair of points \((y_1, x_1), (y_2, x_2)\) on such a line, \( y_2 = y_1 \). The line through \((0, 0)\), and \((1, 1)\) has slope 1. The line through \((0, 0)\) and \((-1, 1)\) has slope \(-1\). The fact that these last two lines are perpendicular follows from the following general result.

**Theorem 1.** Two non-vertical lines are parallel lines if and only if they have the same slope. Two lines, neither of which is vertical, are perpendicular lines if and only if the product of their slopes is \(-1\).

**EXAMPLE:** As an example, we will show that the triangle through the points \((1, 1), (2, 3), \) and \((0, 4)\) is a right triangle: The slope of the line through the first two points is \((3 - 1)/(2 - 1) = 2\). The slope of the line through the second and third points is \((4 - 3)/(0 - 2) = -\frac{1}{2}\). Since these slopes are different, the three points do not lie on the same line, and therefore they form a triangle. Since \(2 \cdot (-\frac{1}{2}) = -1\), two of the sides are perpendicular, so the triangle is a right triangle.

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An infinite number of lines, all parallel, will have the same slope \( m \). There is only one line, however, with a given slope \( m \) going through a given point \( (x_0, y_0) \). If we know the slope \( m \) and a point \( (x_0, y_0) \) on a line, then for any point \( (x, y) \neq (x_0, y_0) \) on the line, \( m = \frac{y - y_0}{x - x_0} \), so \( y = m(x - x_0) + y_0 \).

Here, we have an equation making \( y \) a function of \( x \) for which the graph is the line in question. This equation is called the **Point-Slope Equation** for the line.

A special case of such an equation occurs when \( x_0 = 0 \). In this case, the point \( b = y_0 \) on the \( y \)-axis is called the **\( y \)-intercept** of the line and we have the **Slope-Intercept Equation** for the line \( y = mx + b \).

A line goes through the origin \((0, 0)\) if and only if the line is given by an equation of the form \( y = mx \).

**EXAMPLE**: The line with slope 3 through the point \((1, 2)\) is the graph of the function given by the Point-Slope Equation \( y = 3(x - 1) + 2 = 3x - 1 \). This is also the Slope-Intercept Equation of the line with slope 3 that intercepts the \( y \)-axis at the point \((0, -1)\). If you forget how to do this, you can always start by writing down the slope \( m = 3 = (y - 2)/(x - 1) \).

A horizontal line with \( y \)-intercept \( b \) is given by the equation \( y = b \). Notice that nothing is said about \( x \). This means that \( x \) can take all possible values while the value of \( y \) is fixed at \( b \).

If we reverse the role of \( x \) and \( y \), then we see that a vertical line has the formula \( x = a \), where \( a \) is the \( x \)-intercept of the line; that is, \((a, 0)\) is the point where the line intercepts the \( x \)-axis.

All of these formulas for lines, even vertical lines, have the form \( Ax + By + C = 0 \). Such an equation is called a linear equation provided not both \( A \) and \( B \) are 0. Conversely, any such linear equation is the equation of a line as we see by considering two cases.

**Case 1**: If \( B = 0 \), then \( A \neq 0 \). The equation is then equivalent to the equation of the vertical line \( x = -C/A \), that is, this equation and the original equation are satisfied by the same set of points \((x, y)\). The line is not the graph of a function of \( x \). It is the graph of a function of \( y \).

**Case 2**: If \( B \neq 0 \), then the equation \( Ax + By + C = 0 \) is equivalent to the equation

\[
y = -\frac{A}{B} \cdot x - \frac{C}{B}
\]

for a line with slope \(-A/B\) and \( y \)-intercept \(-C/B\).

**EXAMPLE**: We will find the slope and \( y \)-intercept of the equation \( 3x - 2y + 6 = 0 \). This equation has the same solutions \((x, y)\) as the equation \( y = -(3/2) \cdot x - (6/2) = (3/2)x + 3 \). The slope of this line is \(3/2\) and the \( y \)-intercept is 3.