due Apr 24 before class, answer without justification will receive 0 points. Staple all your papers.

1: Find the number of possible tilings of triangular piece $n \times 2$. Example for $n = 5$.

![Example for n = 5](image)

using the following three kinds of pieces:

![Three kinds of pieces](image)

Another way – suppose that you can cut the tripe of triangles along lines. After the cutting, you are left with pieces that look like a triangle rotated by 180 degrees, triangle or a piece that is a composition of four triangles. The pieces are like this up to translation (no rotation allowed). How many different cuttings are there?

2: Let there be $2n$ points $V$ on a circle in the plane. A perfect matching $M$ is a set of segments with endpoints only from $V$ and every point in $V$ is an endpoint of exactly one segment. Note that $|M| = n$ as one segment needs exactly 2 points from $V$. A matching $M$ in non-crossing if the segments are disjoint. Find the number of non-crossing perfect matchings for $2n$ points.

This can be stated in graph theory language as follows. Count the number of perfect matchings of $K_{2n}$ with vertices are vertices of a regular $2n$-gon in the plane such that the edges of the matching do not cross.

Example for $n = 3$ and hence 6 points.

![Examples for n = 3](image)

3: Find the number of possibilities to build stairs of height $n$ using $n$ rectangular bricks.

All the possibilities for $n = 4$ are depicted.
4: (P. 315, #2) Prove that the number of 2-by-\(n\) arrays
\[
\begin{bmatrix}
  x_{11} & x_{12} & \cdots & x_{1n} \\
  x_{21} & x_{22} & \cdots & x_{2n}
\end{bmatrix}
\]
that can be made from numbers 1, 2, \ldots, 2\(n\) such that
\[
x_{11} < x_{12} < \cdots < x_{1n} \\
\quad x_{21} < x_{22} < \cdots < x_{2n} \\
\quad x_{11} < x_{21}, x_{12} < x_{22}, \ldots, x_{1n} < x_{2n},
\]
equals the \(n\)th Catalan number, \(C_n\).

5: Using the difference sequence method, find a closed form the following sum:
\[
\sum_{k=0}^{n} k^4 - k.
\]

6: (P.316, #7) The general term \(h_n\) of a sequence is a polynomial in \(n\) of degree 3. If the first four entries in the 0th row of its difference table are 1,-1,3,10, determine \(h_n\) and a formula for \(\sum_{k=0}^{n} h_k\).

7: (P.316, #8) Find the sum of the fifth powers of the first \(n\) positive integers.