due Mar 13 before class, answer without justification will receive 0 points. Staple all your papers.

1: (P. 257, #1) Let $f_0, f_1, f_2, \ldots, f_n, \ldots$ denote the Fibonacci sequence. By evaluating each of the following expressions for small values of $n$, conjecture a general formula and then prove it, using mathematical induction and the Fibonacci recurrence:
(a) $f_1 + f_3 + f_5 + \cdots + f_{2n-1}$
(b) $f_0 + f_2 + f_4 + \cdots + f_{2n}$
(c) $f_0 - f_1 + f_2 - f_3 + \cdots + (-1)^n f_n$
(d) $f_0^2 + f_1^2 + f_2^2 + \cdots + f_n^2$

2: (P. 257, #3) Prove the following about the Fibonacci numbers:
(a) $f_n$ is even if and only if $n$ is divisible by 3.
(b) $f_n$ is divisible by 3 if and only if $n$ is divisible by 4.
(c) $f_n$ is divisible by 4 if and only if $n$ is divisible by 6.

3: (P. 258, #11) The Lucas numbers $l_0, l_1, l_2, \ldots, l_n, \ldots$ are defined using the same recurrence relation defining the Fibonacci numbers, but with different initial conditions:
$$l_n = l_{n-1} + l_{n-2}, \quad (n \geq 2), \quad l_0 = 2, \quad l_1 = 1.$$ 
Prove that
(a) $l_n = f_{n-1} + f_{n+1}$ for $n \geq 1$
(b) $l_0^2 + l_1^2 + \cdots + l_n^2 = l_n l_{n+1} + 2$ for $n \geq 0$.

4: Find generating functions for the following sequences:
(a) 0, 0, 0, 6, -6, 6, -6, 6, -6, 6, -6, \ldots
(b) 1, 2, 4, 1, 3, 9, 1, 4, 16, 1, 5, 25, 1, 6, 36, \ldots
(c) $\frac{1}{2}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}, \ldots$

5: For every $n \geq 0$ determine the coefficient at $x^n$ in $(1 + x)^2(1 + x^2)^2(1 + x^4)^2(1 + x^8)^2 \cdots$ which is equal to $\prod_{i=0}^{\infty}(1 + x^{2^i})^2$. 
6: (P. 259, #15) Determine the generating function for the sequence of cubes

\[0, 1, 8, 27, \ldots, n^3, \ldots\]

7: (P. 259, #18) Determine the generating function for the number \(h_n\) of nonnegative integral solutions of

\[2e_1 + 5e_2 + e_3 + 7e_4 = n.\]