due **Feb 27** before class, answer without justification will receive 0 points. Staple all your papers.

1: *P. 156, #29* Find and prove a formula for

\[ \sum_{r,s,t \geq 0 \text{ and } r+s+t=n} \binom{m_1}{r} \binom{m_2}{s} \binom{m_3}{t} \]

where the summation extends over all nonnegative integers \( r, s \) and \( t \) with sum \( r + s + t = n \).

2: *P. 158, #37* Use the multinomial theorem to show that, for positive integers \( n \) and \( t \),

\[ t^n = \sum \binom{n}{n_1 n_2 \ldots n_t} \]

where the summation extends over all nonnegative integral solutions \( n_1, n_2, \ldots, n_t \) of \( n_1 + n_2 + \ldots + n_t = n \).

3: Find
(a) the coefficient of \( x^3y^7 \) in the expansion of \( (2x+y)^{10} \);
(b) the coefficient of \( x^{13}y^{77} \) in the expansion of \( (3x-2y)^{90} \);
(c) the coefficient of \( x_1^3x_2^3x_3x_4^2 \) in the expansion of \((x_1 - x_2 + 2x_3 - 2x_4)^9 \)

4: *P. 158, #44* Prove that,

\[ \sum_{n_1+n_2+n_3=n} \binom{n}{n_1 n_2 n_3} (-1)^{n_1-n_2+n_3} = (-3)^n \]

where the summation extends over all nonnegative integral solutions of \( n_1 + n_2 + n_3 = n \).

5: *P. 160, #50* Consider the partially ordered set \((X, R)\), where \( X = \{1, 2, \ldots, 12\} \) and \( xRy \) if \( y \) is divisible by \( x \), i.e., \( R = \{(x, y) \mid y \text{ is divisible by } x \} \).
(a) Determine a chain of largest size and a partition of \( X \) into the smallest number of antichains.
(b) Then determine an antichain of largest size and a partition of $X$ into the smallest number of chains.

6: (P. 198, #2 ) Find the number of integers between 1 and 10,000 inclusive that are not divisible by 4, 6, 7, or 10.

7: (P. 198, # 6) A bakery sells chocolate, cinnamon, and plain doughnuts and at a particular time has 6 chocolate, 6 cinnamon, and 3 plain. If a box contains 12 doughnuts how many different options are there for a box of doughnuts?

8: (P. 198, # 8.) Determine the number of solutions of the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 14$ in positive integers $x_1, x_2, x_3, x_4$ and $x_5$ not exceeding 5.