Math-484 Homework #6 (Least squares)
I will finish the homework before 10am Oct 17. If I spot a mathematical mistake I will let
the lecturer know as soon as possible.
I will write clearly and neatly as the grader is not an expert in cryptography. I will sign each
paper of my work and indicate if I am C14 (4 hours student).

1: (Can I do least squares solution for not just linear regression?)
Compute best least square fit for polynomial
\[ p(t) = x_0 + x_1 t + x_2 t^2 \]
and data

<table>
<thead>
<tr>
<th>( t_i )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_i )</td>
<td>-5</td>
<td>-1</td>
<td>4</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>-1</td>
</tr>
</tbody>
</table>

2: (Can I compute and use least squares using QR factorization?)
Find best least squares solution to inconsistent linear system using QR factorization.

\[
\begin{pmatrix}
1 & 1 & 2 \\
1 & 4 & 6 \\
1 & 1 & 0 \\
1 & 4 & 4 \\
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
\end{pmatrix}
=
\begin{pmatrix}
4 \\
4 \\
2 \\
2 \\
\end{pmatrix}
\]

3: (What is minimum norm solution?)
Find the minimum norm solution of the underdetermined linear system
\[
2x_1 + x_2 + x_3 + 5x_4 = 8 \\
-x_1 - x_2 + 3x_3 + 2x_4 = 0
\]

4: (What is the projection?)
Find vector \( \mathbf{x} \in \mathbb{R}^3 \) that is closest to \((1, 1, 1)\) where \( \alpha, \beta \in \mathbb{R} \) and
\[
\mathbf{x} = \alpha(1, 1, 2) + \beta(2, -1, 1)
\]

5: (Do I understand definitions?)
Let \( A \) be a matrix with linearly independent columns. Prove that:
\[ a) \quad AA^\dagger = A \]
\[ b) \quad A^\dagger A = (A^\dagger A)^\dagger \]
c) $P_{R(A)}$ is symmetric  

d) $P_{R(A)}^2 = P_{R(A)}$

6: *(Gradient and orthogonal complements. C14 only)*

Let $f(x)$ be a function on $\mathbb{R}^n$ with continuous first partial derivatives and let $M$ be a subspace of $\mathbb{R}^n$. Suppose $x^* \in M$ minimizes $f(x)$ on $M$. Show $\nabla f(x^*) \in M^\perp$.

If, in addition, $f(x)$ is convex, then show that any $x^* \in M$ such that $\nabla f(x^*) \in M^\perp$ is a global minimizer of $f(x)$ on $M$. 