due **Apr 18 strictly before class**
Solutions without explanation will receive no points.

1: Find the number of possibilities to build stairs of height $n$ using $n$ rectangular bricks. All the possibilities for $n = 4$ are depicted.

2: **P. 315, # 2** Prove that the number of 2-by-$n$ arrays

$$
\begin{bmatrix}
  x_{11} & x_{12} & \cdots & x_{1n} \\
  x_{21} & x_{22} & \cdots & x_{2n}
\end{bmatrix}
$$

that can be made from numbers $1, 2, \ldots, 2n$ such that

$x_{11} < x_{12} < \cdots < x_{1n}$

$x_{21} < x_{22} < \cdots < x_{2n}$

$x_{11} < x_{21}, x_{12} < x_{22}, \ldots, x_{1n} < x_{2n},$

equals the $n^{th}$ Catalan number, $C_n$.

3: **P. 316, # 12** Prove that the Stirling numbers of the second $S(n, k)$ kind satisfy the following relations:

(a) $S(n, 1) = 1$ for $n \geq 1$

(b) $S(n, 2) = 2^{n-1} - 1$ for $n \geq 2$

(c) $S(n, n - 1) = \binom{n}{2}$ for $n \geq 1$

(d) $S(n, n - 2) = \binom{n}{3} + 3\binom{n}{4}$ for $n \geq 2$
4: Let \([x]_n = x \cdot (x - 1) \cdot (x - 2) \cdot (x - 3) \cdots (x - n + 1)\) and \(S(n, k)\) be the Stirling number of the second kind. Show that

\[ x^n = \sum_{k=1}^{n} S(n, k)[x]_k. \]

5: P. 317, #16 Compute the Bell number \(B_8\).

6: P. 317, #19 Prove that the Stirling numbers of the first kind satisfy the following formulas:
   (a) \(|s(n, 1)| = (n - 1)!\) for \(n \geq 1\)
   (b) \(|s(n, n - 1)| = \binom{n}{2}\) for \(n \geq 1\)