1: What is the probability of these events when we randomly choose a permutation of \( \{1, 2, \ldots, n\} \) where \( n \geq 4 \)?
   (a) 2 precedes \( n \).
   (b) 1 precedes 2 and 2 precedes \( n \).
   (c) 1 immediately precedes 3.
   (d) 1 immediately precedes 3, and \( n \) immediately precedes 2.

2: Suppose that \( E_1 \) and \( E_2 \) are events such that \( p(E_1) = \frac{3}{4} \) and \( p(E_2) = \frac{4}{5} \). Show that \( \frac{11}{20} \leq p(E_1 \cap E_2) \leq \frac{3}{4} \).

3: A student takes a random card from a deck of 52 cards, looks at it and puts it back. If he does this \( k \) times, what is the probability that he took a card of some kind at least twice? What is the smallest \( k \) such that this probability is greater than \( \frac{1}{2} \)?

4: Suppose that \( E \) and \( F \) are events in a sample space and that \( p(E) = \frac{1}{3} \), \( p(F) = \frac{1}{2} \), and \( p(E|F) = \frac{2}{5} \). Find \( p(F|E) \).

5: When a test for steroids is given to soccer player, 98% of players taking steroids test with positive result and 12% of the players not taking steroids have positive test result. Suppose that 5% of soccer players really take steroids. What is the probability that the soccer player who has a positive test really takes steroids?

6: A space probe near Neptune communicates with Earth using bit strings. Suppose that in its transmissions it sends a 1 one-third to the time and a 0 two-thirds of the time. When a 0 is sent, the probability that it is received correctly is 0.9, and the probability that it is received incorrectly (as a 1) is 0.1. When a 1 is sent, the probability that it is received correctly is 0.8, and the probability that it is received incorrectly (as a 0) is 0.2.
   a) Find the probability that a 0 is received.
b) Use Baye’s theorem to find the probability that a 0 was transmitted given that a 0 was received.

7: Let $h_n$ equal the number of different ways in which the squares of a $1 \times n$ chessboard can be colored using the colors white, red, and blue in such a way that no two squares that are colored red are adjacent. Find and verify a recurrence relation for $h_n$. Then find a formula for $h_n$.

8: Solve the recurrence relation $h_n = 4h_{n-2}, (n \geq 2)$ with initial values $h_0 = 0$ and $h_1 = 1$.

9: Solve the recurrence relation $h_n = 8h_{n-1} - 16h_{n-2}, (n \geq 2)$ with initial values $h_0 = -1$ and $h_1 = 0$. 