1: Use the bubble sort and the insertion sort algorithms to sort 5, 3, 9, 7, 1 showing the lists obtained at each step. (Insertion sort is on page 198, or see Wikipedia.)

2: Determine whether each of these functions is $O(x^3)$ and whether it is $o(x^3)$:

(a) $f_1(x) = 1000 + 10x^2 + \frac{x^3}{1+x}$;
(b) $f_2(x) = -1000 + x^2 + x\frac{x^3}{1+x}$;
(c) $f_3(x) = \frac{2^x}{x^3} - x^3$;
(d) $f_4(x) = 3\log_2 x$.

$(f(x) = o(g(x))$ means that $f(x)$ grows much less than $g(x)$. Precise definition is that for every $c$ exists $x_0$ such that for all $x \geq x_0$ holds $f(x) < cg(x)$. Notice that $f(x) = o(g(x))$ iff $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0$. See Wikipedia or page 218 from the book for more details.

3: For functions $f(x) = \frac{x^2}{3-x} \log_3 x + \sqrt{x}$ and $g(x) = \frac{x^5+x}{1+x+x^4}$, determine whether $f(x) = O(g(x))$, $f(x) = \Omega(g(x))$, or $f(x) = \Theta(g(x))$.

4: Use mathematical induction to prove that for harmonic numbers $H_k = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{k}$, the inequality $H_{2^n} \leq 1 + n$ holds for every nonnegative integer $n$.

5: Use mathematical induction to prove that $4n^3 + 5n$ is divisible by 3 for every nonnegative integer $n$.

6: Use mathematical induction to prove that $n^2 - 1$ is divisible by 8 for every positive odd integer $n$. 