Problem 1

1: Draw the Venn diagrams of:

- $A \cup B$
- $(A \cap B) \setminus C$
- $(\overline{A \cup B \cup C}) \cup (A \cap B \cap C)$
- $(A \setminus B) \cap (C \setminus B)$
2: What is the precise definition of \( f(x) = \omega(g(x)) \)?

Find the smallest integer \( n \) such that \( f(x) = O(x^n) \) where
(a) \( f(x) = 2x^3 + x^2 \log x \)
(b) \( f(x) = 3x^3 + (\log x)^4 \)
(c) \( f(x) = 0.5x^2 + 0.4x^2 \log x \)
(d) \( f(x) = (x^4 + x^2 + 1)/(x^3 + 1) \)
(e) \( f(x) = (x^4 + 5 \log x)/(x^4 + 1) \)

Justify your answer.
3: Write the precise definition what does it mean that a function $f : A \to B$ is one-to-one.

Decide for the following functions if they are surjective, bijective or injective
(a) $f_1(x) = -2x^3$ where $f_1 : \mathbb{R} \to \mathbb{R}$
(b) $f_2(x) = -2x^3$ where $f_2 : \mathbb{Z} \to \mathbb{Z}$
(c) $f_3(x) = (-1)^x \lfloor x/16 \rfloor$ where $f_3 : \mathbb{N} \to \mathbb{Z}$
(d) $f_4(x) = (-1)^x \lfloor (x + 1)/2 \rfloor$ where $f_4 : \mathbb{Z}^+ \to \mathbb{Z}$
4: Draw the graphs of these functions.
(a) \( f_1(x) = \lfloor x - \frac{1}{2} \rfloor + 1 \);
(b) \( f_2(x) = \lfloor 2x + \frac{1}{2} \rfloor - \lceil x - \frac{1}{2} \rceil \);
(c) \( f_3(x) = \lfloor 0.5 \lceil 2x/3 \rceil + 0.5 \rfloor \).
5: Show using mathematical induction that for every integer $n \geq 0$,

$$1 + 3 + 9 + 27 + \cdots + 3^n = \frac{3^{n+1} - 1}{2}.$$
6: Which amounts of money can be formed using just two-dollar bills and five-dollar bills? Prove your answer using strong induction!

(It is not necessary to use at least one from each kind - you may use zero.)
7: (Bonus question) Suppose that \( \log_2 f(x) = O(\log_2 g(x)) \). Show that it does not necessarily mean that \( f(x) = O(g(x)) \).

Describe halting problem. A precise description is required. Write also prove if you know.
Paper for attempts.