Math-484 Homework #6

I will finish this homework before 11 am Oct 12 and bring it to class. If I have troubles with my work I may come to the study session on Oct 10, 5-7 pm, 145 Altgeld Hall. If I spot a mathematical mistake I will let the lecturer know as soon as possible. I will write clearly and neatly as the grader is not an expert in cryptography. I will sign each paper of my work and indicate if I am D14 (4 hours student).

**Exercise 1:** (Can I do least squares solution for not just linear regression?)

Compute best least square fit for polynomial

\[ p(t) = x_0 + x_1 t + x_2 t^2 \]

and data

<table>
<thead>
<tr>
<th>( t_i )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_i )</td>
<td>-5</td>
<td>-1</td>
<td>4</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>-1</td>
</tr>
</tbody>
</table>

**Exercise 2:** (Can I compute and use least squares using QR factorization?)

Find best least squares solution to inconsistent linear system using QR factorization.

\[
\begin{pmatrix}
1 & 1 & 2 \\
1 & 4 & 6 \\
1 & 1 & 0 \\
1 & 4 & 4
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3
\end{pmatrix}
=
\begin{pmatrix}
4 \\
4 \\
2 \\
2
\end{pmatrix}
\]

**Exercise 3:** (What is minimum norm solution?)

Find the minimum norm solution of the underdetermined linear system

\[
\begin{align*}
2x_1 + x_2 + x_3 + 5x_4 &= 8 \\
-x_1 - x_2 + 3x_3 + 2x_4 &= 0
\end{align*}
\]

**Exercise 4:** (What is the projection?)

Find vector \( \mathbf{x} \in \mathbb{R}^3 \) that is closest to \((1,1,1)\) where \( \alpha, \beta \in \mathbb{R} \) and

\[ \mathbf{x} = \alpha(1,1,2) + \beta(2,-1,1) \]

**Exercise 5:** (Do I understand definitions?)

Let \( A \) be a matrix with linearly independent columns. Prove that:

a) \( AA^\dagger A = A \)

b) \( A^\dagger A = (A^\dagger A)^\dagger \)

c) \( P_{R(A)} \) is symmetric

d) \( P_{P_{R(A)}} = P_{R(A)} \)

**Exercise 6:** (Gradient and orthogonal complements. D14 only)

Let \( f(\mathbf{x}) \) be a function on \( \mathbb{R}^n \) with continuous first partial derivatives and let \( M \) be a subspace of \( \mathbb{R}^n \). Suppose \( \mathbf{x}^* \in M \) minimizes \( f(x) \) on \( M \). Show \( \nabla f(\mathbf{x}^*) \in M^\perp \).

If, in addition, \( f(x) \) is convex, then show that any \( \mathbf{x} \in M \) such that \( \nabla f(\mathbf{x}^*) \in M^\perp \) is a global minimizer of \( f(x) \) on \( M \).