Math-484 Homework #5
I will finish this homework before 11 am Oct 5 and bring it to class. If I have troubles with my work I may come to the study session on Oct 3, 5-7 pm, 145 Altgeld Hall. If I spot a mathematical mistake I will let the lecturer know as soon as possible.
I will write clearly and neatly as the grader is not an expert in cryptography. I will sign each paper of my work and indicate if I am D14 (4 hours student).

Exercise 1: (Do I know how to solve inconsistent system?)
Find the least squares solution of the inconsistent linear system of 6 equations:

\[
\begin{align*}
  x_1 + x_2 + x_3 &= 3 \\ x_1 + x_3 &= 2 \\ -7x_1 + 8x_2 &= 0
\end{align*}
\]
\[
\begin{align*}
  x_3 &= 1 \\ 2x_1 + 5x_3 &= 8 \\ x_1 + 2x_2 - x_3 &= 1
\end{align*}
\]

Exercise 2: (Can I do understand linear regression?)
Compute the equation of the linear regression line corresponding to the data on the table below:

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>12</td>
<td>11</td>
<td>8</td>
<td>5</td>
<td>2</td>
<td>-3</td>
</tr>
</tbody>
</table>

Exercise 3: (How much can the solution change?)
Compute and compare solutions of the two systems

\[
\begin{pmatrix}
  1 & 1 & 1 \\
  \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
  \frac{1}{4} & \frac{1}{4} & \frac{1}{4}
\end{pmatrix}
\begin{pmatrix}
  x \\
  y \\
  z
\end{pmatrix} =
\begin{pmatrix}
  0 \\
  0 \\
  0
\end{pmatrix}
\quad \text{and} \quad
\begin{pmatrix}
  1 & 1 & 1 \\
  \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
  \frac{1}{4} & \frac{1}{4} & \frac{1}{4}
\end{pmatrix}
\begin{pmatrix}
  x \\
  y \\
  z
\end{pmatrix} =
\begin{pmatrix}
  0 \\
  0 \\
  0.01
\end{pmatrix}.
\]

Exercise 4: (How do I compute generalized inverz?)
Compute generalized inverz \(A^\dagger\) of

\[
A = \begin{pmatrix}
  0 & 1 \\
  1 & 2 \\
  2 & 3
\end{pmatrix}
\]

Exercise 5: (Am I familiar with Gram-Schmidt Process?)
Find orthonormal bases of linear subspaces generated by:
\[a) \quad L_1 = \{(0, 3, 4, 0)^T, (0, 0, 5, 0)^T, (2, 1, 0, 2)^T\} \]
\[b) \quad L_2 = \{(2, 0, 1, 2)^T, (4, 3, 2, 4)^T, (6, -5, 3, 6)^T, (-4, 2, 4, 2)^T\} \]

Exercise 6: (Applications of \((A - G)\). D14 only)
Solve the following classical calculus problems by making use of \((A - G)\) inequality.
\[a) \quad \text{Find the largest circular cylinder that can be inscribed in a sphere of a given radius.} \]
\[b) \quad \text{Find the smallest radius } r \text{ such that a circular cylinder of volume 8 cubic units can be inscribed in the sphere of radius } r. \]