Math-484 Homework #1
I will finish the homework before 11 am Aug 31 and bring it to class. If I have troubles with my work I may come to the study session on Aug 29, 5-7 pm, 145 Altgeld Hall. If I spot a mathematical mistake I will let the lecturer know as soon as possible.
I will write clearly and neatly as the grader is not an expert in cryptography. I will sign each paper of my work and indicate if I am D14 (4 hours student).

Exercise 1: (I will check if I can use Theorems 1,2 and 3)
Find the local and global minimizers and maximizers of the following functions:
(a) $f(x) = x^2 + 2x$
(b) $f(x) = x^2 e^{-x^2}$

Exercise 2: (I will recall few basic definitions)
Determine the dimension of the smallest subspace of $\mathbb{R}^4$ that contains vectors $(0, 1, 0, 1)$, $(3, 4, 1, 2)$, $(6, 4, 2, 0)$ and $(-3, 1, -1, 3)$.

Exercise 3: (I will recall what are determinants)
Compute determinants of the following real matrices:
(a) \[
\begin{pmatrix}
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0
\end{pmatrix}
\]
(b) \[
\begin{pmatrix}
0 & -2 & 1 & 0 \\
4 & a & b & 1 \\
1 & c & d & 4 \\
0 & 1 & -2 & 0
\end{pmatrix}
\] where $a, b, c, d \in \mathbb{R}$ are parameters

Exercise 4: (I will recall what are eigenvalues and eigenvectors)
Compute eigenvalues and eigenvectors of the following real matrix
\[
\begin{pmatrix}
2 & 6 \\
6 & -3
\end{pmatrix}
\]

Exercise 5: (I will check the definition of semidefinite and recall computing with matrices and vectors.)
Suppose that $A$ is a square matrix and suppose that there is another matrix $B$ such that $A = B^T B$. Show that $A$ is positive semidefinite.

Hint:
Recall that $y \cdot B^T x = By \cdot x$

Exercise 6: (I will check the definition of semidefinite more closely. D14 only)
Suppose that $A$ is a $n \times n$-symmetric matrix for which $a_{ii}a_{jj} - a_{ij}^2 < 0$ for some $i \neq j$. Show that $A$ is indefinite.

Hint:
See (1.3.4)(c) in the textbook.