This is not exam - it just contains some list of definitions and theorems. It will be updated over time. If you see some a mistake or a typo, please let me know.
This version is from: 12:12 December 12, 2011. It should contain all theoretical questions. Of course, there will be many computational questions on the exam too!

Question 1:
Write definitions of the following terms:
- (global,local)(strict) minimizer and maximizer of a real function page 2
- critical point of a real function page 2
- cosine of two vectors page 6
- ball $B(x, r)$ (what is $x$ and $r$?) page 6
- interior $D^0$ of set $D \subseteq \mathbb{R}^n$ page 6, page 164
- open set $D \subseteq \mathbb{R}^n$ page 6
- closed set $D \subseteq \mathbb{R}^n$ page 7
- compact set $D \subseteq \mathbb{R}^n$ page 6
- (global,local)(strict) minimizer and maximizer of a function $f : \mathbb{R}^n \to \mathbb{R}$ page 8
- critical point of a function $f : \mathbb{R}^n \to \mathbb{R}$ page 8
- gradient $\nabla f(x)$ where $f : \mathbb{R}^n \to \mathbb{R}$ page 10
- Hessian $Hf(x)$ where $f : \mathbb{R}^n \to \mathbb{R}$ page 10
- quadratic form associated with a matrix $A$ page 12
- (positive,negative)(semi)definite matrix page 13
- indefinite matrix page 13
- $\Delta_k$, the $k$th principal minor of a matrix $A$ page 16
- $f : \mathbb{R}^n \to \mathbb{R}$ being coercive page 25
- eigenvalues and eigenvectors of a matrix $A$ page 29
- $C \subseteq \mathbb{R}^n$ being convex page 38
- closed and open half-spaces in $\mathbb{R}^n$ page 40
- convex combination of $k$ vectors from $\mathbb{R}^n$ 41
- convex hull of $D \subseteq \mathbb{R}^n$ 42
- (strictly) convex and concave function $f : C \to \mathbb{R}$, where $C \subseteq \mathbb{R}^n$ page 49
- posynomial page 67
- primal and dual geometric program page 67,68
- best least squares $k$th degree polynomial page 135
- linear regression line page 135
- best least squares solution of (inconsistent) linear system page 136
- generalized inverse of a matrix $A$ page 136
- inconsistent linear system page 136–137
- orthonormal vectors page 138
- subspace of $\mathbb{R}^n$ page 141
- orthogonal complement of a subspace of $\mathbb{R}^n$ page 142
- $P_M$ - orthogonal projection of $\mathbb{R}^n$ onto $M$ page 144
- underdetermined system of linear equations page 145
- $H$-inner product page 149
- $H$-norm page 149
- $H$-orthogonal vectors  page 149
- $H$-orthogonal complement  page 149
- $H$-generalized inverse page 150
- hyperplane $H$ in $\mathbb{R}^n$ page 158
- boundary point of $C \subset \mathbb{R}^n$ page 158
- closure $\bar{A}$ of $A \subset \mathbb{R}^n$ page 163
- subgradient of $f : \mathbb{R}^n \to \mathbb{R}$ page 168
- subdifferential of $f : \mathbb{R}^n \to \mathbb{R}$ page 168
- feasible vector of a program $(P)$ page 169
- feasible region of a program $(P)$ page 169
- consistent program $(P)$ page 169
- superconsistent program $(P)$ page 169
- convex program and dual convex program pages 169, 200, 201
- supremum of a real valued function defined on $C \subseteq \mathbb{R}^n$ page 170
- infimum of a real valued function defined on $C \subseteq \mathbb{R}^n$ page 170
- $MP$ for program $(P)$ - also define $(P)$ page 171
- $MP(z)$ for program $(P(z))$ - also define $(P(z))$ page 171
- sensitivity vector of a program $(P)$ page 177
- Lagrangian $L(x, \lambda)$ of a program $(P)$ - also define $(P)$ page 182
- complementary slackness conditions - also define $(P)$ page 184
- constrained geometric program $(GP)$ and its dual $(DGP)$ page 193
- linear program $(LP)$ and its dual $(DLP)$ pages 173, 201, 202
- duality gap page 209
- Absolute value penalty function page 217
- penalty parameter page 217
- Courant-Beltrami penalty function page 219
- generalized penalty function page 223
- exact penalty function page 226
- program $(P^*)$ page 230
- $Tr(A)$ - a trace of a matrix $A$ [SDP notes]
- primal and dual form of a semidefinite program $(SDP)$ [SDP notes]
Question 2:
State theorems which give answers to the following questions: *(without proofs)*

- What are the implications of first and second derivatives on minimizers and maximizers of \( f : \mathbb{R} \to \mathbb{R} \)? *Theorem 1.1.5*
- What are the implications of first and second partial derivatives on minimizers and maximizers of a function \( f : \mathbb{R}^n \to \mathbb{R} \)? *Theorem 1.2.5*
- What are implications of definiteness \( Hf(x) \) on global minimizers and maximizers? *Theorem 1.2.9*
- What are implications of definiteness \( Hf(x) \) on local minimizers and maximizers? *Theorem 1.3.6*
- What can you say about extremes of a coercive function? *Theorem 1.4.4*
- What is the relationship between eigenvalues and positive(negative)(semi)definiteness of a symmetric matrix \( A \)? *Theorem 1.5.1*
- What is the relation between convex hull and set of all convex combinations of vectors from \( D \subseteq \mathbb{R}^n \)? *Theorem 2.1.4*
- What do you know about minimizers of (strictly) convex functions (in \( \mathbb{R}^n \))? *Theorem 2.3.4, Corollary 2.3.6*
- Is there relationship between begin s (strictly) convex function and having continuous first partial derivatives (in \( \mathbb{R}^n \))? *Theorem 2.3.5*
- Is there relationship between begin (strictly) convex function and having continuous second partial derivatives (condition using \( Hf(x) \)) (in \( \mathbb{R}^n \))? *Theorem 2.3.7*
- State Arithmetic-Geometric Mean Inequality. Include also when it is equality! *Theorem 2.4.1*
- State duality theorem for geometric programs. *Theorem 2.5.2*
- What does and how to compute \( P_M \)? (orthogonal projection of \( \mathbb{R}^n \) onto \( M \)) *Theorem 4.2.5*
- What is the form of solutions of underdetermined systems? *Theorem 4.3.1*
- What is the form of minimum norm solutions of underdetermined systems? *Theorem 4.3.2*
- What is the form of minimum \( H \)-norm solutions of underdetermined systems? *Theorem 4.4.2*
- What is the way of computing of the closest vector of a convex set to a given vector? *Theorem 5.1.1*
- What is the characterization of the closest vector of a convex set to a given vector using orthogonal complement? *Theorem 5.1.2*
- What is a sufficient condition for existence of a closest vector from a set \( C \) to a given vector \( x \)? *Theorem 5.1.3*
- What is a sufficient condition for existence of a unique closest vector from a set \( C \) to a given vector \( x \)? *Corollary 5.1.4*
- State basic separation theorem. *Theorem 5.1.5*
- State Support theorem. *Theorem 5.1.9*
- What can you say about \( MP(z) \) if \( (P) \) is super consistent? *(Theorem 5.2.6)*
- Can \( MP \) be computed from the sensitivity vector? *(Theorem 5.2.11)*
- State Karush-Kuhn-Tucker Theorem (Saddle point version) *Theorem 5.2.13*
- State Karush-Kuhn-Tucker Theorem (Gradient form) *Theorem 5.2.14*
- State Extended Arithmetic-Geometric Mean Inequality Include also when it is equality! *Theorem 5.3.1*
- What are sufficient condition for a constrained geometric program \((GP)\) to have no duality gap? *Theorem 5.3.5*
- State the duality theorem of linear programming *page 203*
- State duality theorem of convex programming *Theorem 5.4.6*
- What are sufficient conditions for a constrained convex program \((P)\) to have no duality gap? *Theorem 6.3.1*
- State the duality theorem for semidefinite programming *SDP notes - first theorem*
- What do you know about solvability of SDP? *SDP notes - second theorem*
Question 3:
Answer the following:
- Is product of two convex functions convex?
- How to express best least square solution using QR factorization? What is advantage of QR over using generalized inverse?
- Describe the intuition using angles behind the Theorem stating what is the closest vector from a convex set to a given vector. Theorem 5.1.1, page 159
- Does every convex set always contain a vector that is closest to a given vector? Why?
- Is MP(z) always continuous?
- Why do we consider MP and MD instead of min and max?
- What is relation between KKT multipliers and sensitivity vector?
- Why is it necessary to use extended (AG) for bounding \( g_i(t) \) while solving constrained geometric programs?
- How can you (in theory) try to solve geometric program using its dual? (page 201) - What is a disadvantage of Courant-Beltrami penalty function? Answer: "isn't exact" - but more details are expected ;-)
- What are advantages (or consequences) of having no duality gap? Answer: "primal-dual algorithm, algorithms with guaranteed performance, certificate of optimality" - but more details are expected - like what is what ;-)?
- Describe how is it possible to change the objective function of a convex program such that the objective is coercive. What is the reason for doing it? pages 229,230
- Can be ANY linear program expressed as SDP?
- Why is semidefinite programming important? Answer hints: What can you express as SDP? Can you solve it? - Give example how can you use the condition that a matrix is positive semidefinite while trying to express a problem as semidefinite program. Answer: You can use "quadratic constraint" and write express it as \( x^T y - z^2 \) and this "corresponds" to determinant of a \( 2 \times 2 \) matrix. Or if you can express your variables as vectors, you may use that every positive semidefinite matrix \( A \) has a unique decomposition \( A = U^T U \). See SDP notes for both of these two. Maybe you can give a small example.
Question 4: [Computational question on SDP will look like this:]
Express the following program as a semidefinite program. *(Do not solve the resulting program.)* See SDP notes for examples and your notes for *(LP) → (SDP)* example.
Question X: D14 only (D13 may try too if they wish)
- State and prove the theorem relating local and global minimizers of a convex function (in \( \mathbb{R}^n \)). Theorem 2.3.4
- State and prove Arithmetic-Geometric Mean Inequality. Include also when it is equality. Theorem 2.4.1
- State and prove basic separation theorem. Theorem 5.1.5
- State and prove Support theorem. Theorem 5.1.9
- Derive dual geometric program from primal using AG inequality. page 67-68
- State and prove Karush-Kuhn-Tucker Theorem in Saddle point version. Theorem 5.2.13
- State and prove extended arithmetic geometric mean inequality. Theorem 5.3.1