I. Find all vertical and horizontal asymptotes of the graph of the given function.

1. \( f(x) = \frac{x^2 + 17x + 5}{x^2 + 8x + 12} \)

2. \( f(x) = \frac{3x^2 + 17x + 5}{4x^2 + 8x - 12} \)

3. \( f(x) = \frac{3x^2 + 17x + 5}{x^3 + 5x^2 + 6x} \)

4. \( f(x) = \frac{3x^2}{\sqrt{9x^4 + 1}} \)

5. \( f(x) = \frac{3x^3}{\sqrt{4x^6 + 1}} \)

6. \( f(x) = \frac{5x^2}{\sqrt{x^4 - 16}} \)

II. Find the absolute maximum and minimum of the given function on the specified interval.

1. \( f(x) = x^2 - 5x + 6; \quad -3 \leq x \leq -1 \)

2. \( f(x) = x^2 - 5x + 6; \quad 0 \leq x \leq 5 \)

3. \( f(x) = \frac{x^3}{3} - 36x + 6; \quad 0 \leq x \leq 1 \)

4. \( f(x) = \frac{x^3}{3} - x + 6; \quad 0 \leq x \leq 2 \)

5. \( f(x) = \frac{1}{x^2 - 9}; \quad 0 \leq x \leq 2 \)
6. \( f(x) = \frac{1}{x^2}; \quad x > 0 \)

7. \( f(x) = x + \frac{2}{x}; \quad 1/2 \leq x \leq 3 \)

8. \( f(x) = (x^2 - 9)^5; \quad -3 \leq x \leq 2 \)

III. Find where \( P(q) \) is maximized and \( A(q) \) is minimized.

1. \( p(q) = 37 - 2q; \quad C(q) = 3q^2 + 5q + 75 \)
I. Find all vertical and horizontal asymptotes of the graph of the given function.

1. \( f(x) = \frac{x^2 + 17x + 5}{x^2 + 8x + 12} \)
   Answer:
   V. A.:
   \( x^2 + 8x + 12 = 0 \) when \( x = -2, -6 \)
   Vertical Asymptotes: \( x = -6, -2 \)

   H.A.:
   \[ \lim_{x \to \infty} \frac{x^2 + 17x + 5}{x^2 + 8x + 12} = 1 \]
   \[ \lim_{x \to -\infty} \frac{x^2 + 17x + 5}{x^2 + 8x + 12} = 1 \]
   Since the polynomials in the numerator and denominator have the same degree the limits are equal to the ratio of the leading coefficients. Horizontal asymptotes: \( y = 1 \)

2. \( f(x) = \frac{3x^2 + 17x + 5}{4x^2 + 8x - 12} \)
   Answer:
   V. A.:
   \( 4x^2 + 8x - 12 = 4(x^2 + 2x - 3) = 4(x + 3)(x - 1) = 0 \) when \( x = -3, 1 \)
   Vertical Asymptotes: \( x = -3, 1 \)

   H.A.:
   \[ \lim_{x \to \infty} \frac{3x^2 + 17x + 5}{4x^2 + 8x - 12} = \frac{3}{4} \]
   \[ \lim_{x \to -\infty} \frac{3x^2 + 17x + 5}{4x^2 + 8x - 12} = \frac{3}{4} \]
   Since the polynomials in the numerator and denominator have the same degree the limits are equal to the ratio of the leading coefficients. Horizontal asymptotes: \( y = \frac{3}{4} \)

3. \( f(x) = \frac{3x^2 + 17x + 5}{x^3 + 5x^2 + 6x} \)
   Answer:
   V. A.:
   \( x^3 + 5x^2 + 6x = x(x^2 + 5x + 6) = x(x + 3)(x + 2) = 0 \) when \( x = -3, -2 \)
   Vertical Asymptotes: \( x = -3, -2 \)
H.A.:
\[ \lim_{x \to \infty} \frac{3x^2 + 17x + 5}{x^3 + 5x^2 + 6x} = 3 \]
\[ \lim_{x \to -\infty} \frac{3x^2 + 17x + 5}{x^3 + 5x^2 + 6x} = 3 \]
Since the polynomials in the numerator and denominator have the same degree the limits are equal to the ratio of the leading coefficients. Horizontal asymptotes: \( y = 3 \)

4. \( f(x) = \frac{3x^2}{\sqrt{9x^4 + 1}} \)
Answer:
V. A.:
\( \sqrt{9x^4 + 1} > 0 \) since \( 9x^4 + 1 > 0 \) for all \( x \).
Vertical Asymptotes: none

H.A.:
\[ \lim_{x \to \infty} \frac{3x^2}{\sqrt{9x^4 + 1}} = \lim_{x \to \infty} \frac{3x^2}{\sqrt{9x^4 + 1}} \frac{1}{\sqrt{x^4}} \]
\[ = \lim_{x \to \infty} \frac{3x^2}{\sqrt{9x^4 + 1}} \frac{1}{\sqrt{x^4}} \]
\[ = \lim_{x \to \infty} \frac{3x^2}{\sqrt{9x^4 + 1}} \frac{1}{\sqrt{x^4}} \]
\[ = \lim_{x \to \infty} \frac{3}{\sqrt{9 + \frac{1}{x^4}}} \]
\[ = \frac{3}{\sqrt{9}} \]
\[ = 3 \sqrt{9} \]
\[ = 3 \]

\[ \lim_{x \to -\infty} \frac{3x^2 + 17x + 5}{x^3 + 5x^2 + 6x} = \lim_{x \to -\infty} \frac{3x^2 + 17x + 5}{x^3 + 5x^2 + 6x} \frac{1}{\sqrt{x^4}} \]
\[ = \lim_{x \to -\infty} \frac{3x^2 + 17x + 5}{x^3 + 5x^2 + 6x} \frac{1}{\sqrt{x^4}} \]
\[ = \lim_{x \to -\infty} \frac{3x^2 + 17x + 5}{x^3 + 5x^2 + 6x} \frac{1}{\sqrt{x^4}} \]
\[ = \lim_{x \to -\infty} \frac{3}{\sqrt{9 + \frac{1}{x^4}}} \]
\[ = \frac{3}{\sqrt{9}} \]
\[ = 1 \]
Horizontal asymptotes: $y = 1$

5. $f(x) = \frac{3x^3}{\sqrt{4x^6 + 1}}$

   Answer:
   
   V. A. :
   
   $\sqrt{4x^6 + 1} > 0$ since $4x^6 + 1 > 0$ for all $x$.
   
   Vertical Asymptotes: none

   H.A.:

   \[
   \lim_{x \to \infty} \frac{3x^3}{\sqrt{4x^6 + 1}} = \lim_{x \to \infty} \frac{3x^3}{\sqrt{4x^6 + 1}}
   \]

   \[
   = \lim_{x \to \infty} \frac{3x^3}{\sqrt{4x^6 + 1}}
   \]

   \[
   = \lim_{x \to \infty} \frac{3x^3}{\sqrt{4x^6 + 1}}
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   \[
   = \lim_{x \to \infty} \frac{3x^3}{\sqrt{4x^6 + 1}}
   \]

   \[
   = \lim_{x \to \infty} \frac{3x^3}{\sqrt{4x^6 + 1}}
   \]

   \[
   = \lim_{x \to \infty} \frac{3x^3}{\sqrt{4x^6 + 1}}
   \]

   \[
   = \frac{3}{\sqrt{4}}
   \]

   \[
   = \frac{3}{2}
   \]

   Horizontal asymptotes: $y = 3/2, -3/2$

6. $f(x) = \frac{5x^2}{\sqrt{x^4 - 16}}$

   Answer:

   V. A. :

   Horizontal asymptotes: $y = 3/2, -3/2$
\[
\sqrt{x^4 - 16} = 0 \text{ when } x^4 - 16 = 0 \text{ when } x = -2, 2.
\]
Vertical Asymptotes: \( x = -2, 2 \)

H.A.:

\[
\lim_{x \to -\infty} \frac{5x^2}{\sqrt{x^4 - 16}} = \lim_{x \to -\infty} \frac{5x^2 - \frac{1}{x^4}}{\sqrt{x^4 - 16}} = \lim_{x \to -\infty} \frac{5x^2 - \frac{1}{x^4}}{\sqrt{x^4}} = \lim_{x \to -\infty} \frac{5x^2}{\sqrt{x^4}} = \frac{5}{\sqrt{1}} = 5
\]

\[
\lim_{x \to +\infty} \frac{5x^2}{\sqrt{x^4 - 16}} = \lim_{x \to +\infty} \frac{5x^2 - \frac{1}{x^4}}{\sqrt{x^4 - 16}} = \lim_{x \to +\infty} \frac{5x^2 - \frac{1}{x^4}}{\sqrt{x^4}} = \lim_{x \to +\infty} \frac{5x^2}{\sqrt{x^4}} = \frac{5}{\sqrt{1}} = 5
\]

II. Find the absolute maximum and minimum of the given function on the specified interval.

1. \( f(x) = x^2 - 5x + 6; \quad -3 \leq x \leq -1 \)

   Answer:
   \[
f'(x) = 2x - 5
   \]
   \[
f'(x) = 0 \text{ when } x = 2.5
   \]
   \( x = 2.5 \) is not in the range, so we don’t need to consider it.
   \[
f(-3) = 30
   \]
   \[
f(-1) = 12
   \]
   The absolute minimum is \( f(-1) = 12 \) and the absolute maximum is \( f(-3) = 30 \).
2. \( f(x) = x^2 - 5x + 6; \ 0 \leq x \leq 5 \)
Answer:
\( f'(x) = 2x - 5 \)
\( f'(x) = 0 \) when \( x = 2.5 \)
\( f''(x) = 2 > 0 \) (\( f(x) \) is concave up at \( x = 2.5 \) ) There is a minimum at \( x = 2.5 \)
\( f(2.5) = -0.25 \)
\( f(0) = 6 \)
\( f(5) = 6 \)
The absolute minimum is \( f(2.5) = -0.25 \) and the absolute maximum is \( f(0) = 6 \).

3. \( f(x) = \frac{x^3}{3} - 36x + 6; \ 0 \leq x \leq 1 \)
Answer:
\( f'(x) = x^2 - 36 \)
\( f'(x) = 0 \) when \( x = -6, 6 \)
Neither of the critical numbers is in the range. So we just have to check the endpoints.
\( f(0) = 6 \)
\( f(1) = -89/3 \)
The absolute minimum is \( f(1) = -89/3 \) and the absolute maximum is \( f(0) = 6 \).

4. \( f(x) = \frac{x^3}{3} - x + 6; \ 0 \leq x \leq 2 \)
Answer:
\( f'(x) = x^2 - 1 \)
\( f'(x) = 0 \) when \( x = -1, 1 \)
\( x = 1 \) is the only critical number in the range.
\( f''(x) = 2x \) and \( f''(1) = 2 > 0 \). There is a relative minimum at \( x = 1 \).
\( f(1) = 16/3 \)
\( f(0) = 6 \)
\( f(2) = 23/3 \)
The absolute minimum is \( f(1) = 16/3 \) and the absolute maximum is \( f(2) = 23/3 \).

5. \( f(x) = 1/(x^2 - 9); \quad 0 \leq x \leq 2 \)
   Answer:
   \[
   f'(x) = -2x/(x^2 - 9)^2
   \]
   \( f'(x) = 0 \) when \( x = 0 \)
   \( x = 0 \) is in the range.
   \[
   f''(x) = \frac{[-2(x^2 - 9)^2 - 2x(2(x^2 - 9)2x)]/(x^2 - 9)^4}{\text{and } f''(0) > 0}. \text{ There is a relative minimum at } x = 0.
   \]
   \( f(0) = -1/9 \)
   \( f(2) = 1/16 \)
   The absolute minimum is \( f(0) = -1/9 \) and the absolute maximum is \( f(2) = 1/16 \).

6. \( f(x) = 1/x^2; \quad x > 0 \)
   Answer:
   \[
   f'(x) = -2/x^3
   \]
   The only critical number is at \( x = 0 \) and \( f(0) \) does not exist.
   Since the function is not defined on a closed interval the extreme value property does not apply.
   Hence since the interval is not closed \( f(x) \) and there are no critical points in the interval, \( f(x) \) does not have an absolute maximum or minimum.

7. \( f(x) = x + 2/x; \quad 1/2 \leq x \leq 3 \)
   Answer:
   \[
   f'(x) = 1 - 2/x^2
   \]
   \( f'(x) = 0 \) when \( x = -\sqrt{2}, \sqrt{2} \)
   The only critical number in the range is \( \sqrt{2} \).
   \[
   f''(x) = 4/x^3
   \]
   \( f''(\sqrt{2}) > 0 \)
   There is a relative minimum at \( x = \sqrt{2} \)
\[ f(\sqrt{2}) = \sqrt{2} + \sqrt{2} = 2\sqrt{2} \]
\[ f(1/2) = 9/2 \]
\[ f(3) = 11/9 \]
The absolute minimum is \( f(\sqrt{2}) = 2\sqrt{2} \) and the absolute maximum is \( f(1/2) = 9/2 \).

8. \( f(x) = (x^2 - 9)^5; \quad -3 \leq x \leq 2 \)
   Answer:
   \[ f'(x) = 5(x^2 - 9)^4(2x) \]
   \[ f'(x) = 0 \text{ when } x = -3, 0, 3 \]
The only critical numbers in the range are \( x = -3, 0 \)
\[ f'(x) < 0 \text{ for } x < -3 \text{ and } -3 < x < 0 \text{ and } f'(x) > 0 \text{ for } 0 < x < 2. \]
There is a relative minimum at \( x = 0 \).
\[ f(0) = -59049 \]
\[ f(-3) = 0 \]
\[ f(2) = -3125 \]
The absolute minimum is \( f(0) = -59049 \) and the absolute maximum is \( f(-3) = 0 \).

III. Find were \( P(q) \) is maximized and \( A(q) \) is minimized.

1. \( p(q) = 37 - 2q; \quad C(q) = 3q^2 + 5q + 75 \)
   Answer:
   \[ P(q) = R(q) - C(q) \]
   Using the marginal analysis criteriion for maximum profit, we need to find where:
   \[ R'(q) = C'(q) \text{ and } R''(q) < C''(q) \]
   \[ R(q) = qp(q) = 37q - 2q^2 \]
   \[ R(q) = 37q - 2q^2 \quad C(q) = 3q^2 + 5q + 75 \]
   \[ R'(q) = 37 - 4q \quad C'(q) = 6q + 5 \]
   \[ R''(q) = -4 \quad C''(q) = 6 \]
Now set \( R'(q) = C'(q) \)
\[
\begin{align*}
37 - 4q &= 6q + 5 \\
32 - 4q &= 6q \\
34 &= 10q \\
q &= 3.4
\end{align*}
\]

When \( q = 3.4 \), \( R'(q) = C'(q) \) and \( R''(q) < C''(q) \). Therefore \( P(q) \) is maximized at \( q = 3.4 \).

\( A(q) = C(q)/q \)

Using the marginal analysis criterion for minimal average cost, we need to find where:

\( A(q) = C'(q) \).

\[
\begin{align*}
A(q) &= C(q)/q = (3q^2 + 5q + 75)/q = 3q + 5 + 75/q \\
C'(q) &= 6q + 5
\end{align*}
\]

Now set \( A(q) = C'(q) \).

\[
\begin{align*}
3q + 5 + 75/q &= 6q + 5 \\
3q + 75/q &= 6q \\
75/q &= 3q \\
75 &= 3q^2 \\
25 &= q^2 \\
q &= -5, 5
\end{align*}
\]

However, by convention, \( q \) is positive.

Therefore \( A(q) \) is minimized when \( q = 5 \).