I. A ball was thrown straight up in the air on Mars with a speed of 20 ft/s. If the acceleration due to gravity on Mars is 10 ft/s², when will the ball hit the ground?

II. What constant acceleration is required to increase the speed of a car from 35 mi/h to 55 mi/h in 5 seconds?

III. What is the constant acceleration needed to change the speed of a car from \( v_0 \) mi/h to \( v_1 \) in \( t \) seconds?

IV. The acceleration of a car in m/s² is given by:

\[
a(t) = \begin{cases} 
0.6t & \text{if } 0 \leq t \leq 10 \\
0 & \text{if } t > 10.
\end{cases}
\]

If the car is stopped at \( t = 0 \), how long will it take to travel 1 km?

V. A car is traveling at 60 mi/h when the brakes are fully applied, producing a constant deceleration of 15 ft/s². What is the distance traveled before the car comes to a stop?

VI. The speed of a runner increased steadily during 3 seconds of a race. Her speeds are given below. Find lower and upper estimates for the distance that she traveled during these three seconds.

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VII. Use the form of the definition of the integral given in Theorem 4 to evaluate the integral.

1. \[\int_0^3 5dx\]

2. \[\int_0^2 (3 - 2x - 3x^2)dx\]

3. \[\int_a^b 5dx\]

4. \[\int_a^b xdx\]
5. \[ \int_{a}^{b} x^2 \, dx \]

6. \[ \int_{a}^{b} x^3 \, dx \]

7. \[ \int_{a}^{b} (cx^3 + dx^2 + ex + f) \, dx \]

VIII. Evaluate the following integrals

1. \[ \int_{-\pi/2}^{\pi/2} \sin(x) + \cos(2x) \, dx \]

2. \[ \int_{-2}^{0} (2 + \sqrt{4 - x^2}) + \sqrt{4 - (x + 2)^2} \, dx \]

3. \[ \int_{-1}^{2} |x + 1| \, dx \]

4. \[ \int_{-1}^{2} |x + 1| - 2|x - 1| \, dx \]

5. \[ \int_{0}^{2} |x^2 + 1| \, dx \]
1 Solutions

I. A ball was thrown straight up in the air from the ground on Mars with a speed of 20 ft/s. If the acceleration due to gravity on Mars is 10 ft/s², when will the ball hit the ground?

Answer:
We assume that up is positive.
\[ a(t) = -10 \]
\[ v(t) = -10t + v_0 \]
From the problem we have \( v_0 = 20 \)
\[ s(t) = -5t^2 + 20t + s_0 \]
Since the ball starts on the ground \( s_0 = 0 \)
\[ s(t) = -5t^2 + 20t = 5t(4 - t) \]
It will hit the ground when \( s(t) = 0 \) when \( t = 4 \).
The ball will hit the ground 4 seconds after it is thrown.

II. What constant acceleration is required to increase the speed of a car from 35 mi/h to 55 mi/h in 5 seconds?

Answer:
5 seconds = \( \frac{5}{3600} \) hours
We assume acceleration is constant \( a(t) = a \)
\[ v(t) = at + v_0 \]
We’re given \( v_0 = 35 \).
\[ v(t) = at + 35 \]
\[ v\left(\frac{5}{3600}\right) = a\frac{5}{3600} + 35 \]
\[ 55 = a\frac{5}{3600} + 35 \]
\[ 20 = a\frac{5}{3600} \]
\[ 14400 = a \]
The car must accelerate at 14400 mi/h² or 4 mi/h/s.

III. What is the constant acceleration needed to change the speed of a car from \( v_0 \) mi/h to \( v_1 \) in \( t \) seconds?

Answer:
\[ v(t) = at + v_0 \]
\[
\begin{align*}
v_1 &= at + v_0 \\
v_1 - v_0 &= at \\
v_1 - v_0 &= t
\end{align*}
\]

It will take \( \frac{v_1 - v_0}{a} \) hours and therefore \( \frac{(v_1 - v_0)3600}{a} \) seconds.

IV. The acceleration of a car in m/s\(^2\) is given by:

\[
a(t) = \begin{cases} 
0.6t & \text{if } 0 \leq t \leq 10 \\
0 & \text{if } t > 10.
\end{cases}
\]

If the car is stopped at \( t = 0 \), how long will it take to travel 1 km?

Answer:

\[
v(t) = \begin{cases} 
0.3t^2 + C_1 & \text{if } 0 \leq t \leq 10 \\
C_2 & \text{if } t > 10.
\end{cases}
\]

Since the car is stopped at \( t = 0 \), we have \( v(0) = 0 \) which gives us \( C_1 = 0 \). Thus

\[
v(t) = \begin{cases} 
0.3t^2 & \text{if } 0 \leq t \leq 10 \\
30 & \text{if } t > 10.
\end{cases}
\]

Velocity is continuous therefore.

\[
\lim_{t \to 10^-} 0.3t^2 = \lim_{t \to 10^+} C_2 \\
30 = c_2
\]

Thus

\[
v(t) = \begin{cases} 
0.3t^2 & \text{if } 0 \leq t \leq 10 \\
30 & \text{if } t > 10.
\end{cases}
\]

Now we find position.

\[
s(t) = \begin{cases} 
0.1t^3 + C_1 & \text{if } 0 \leq t \leq 10 \\
30t + C_2 & \text{if } t > 10.
\end{cases}
\]
We assume \( s(0) = 0 \) which gives us \( C_1 = 0 \) Position is continuous which gives us.

\[
\lim_{t \to 10^-} 0.1t^3 = \lim_{t \to 10^+} 30t + C_2 \\
100 = 300 + C_2 \\
-200 = C_2
\]

\[
s(t) = \begin{cases} 
0.1t^3 & \text{if } 0 \leq t \leq 10 \\
30t - 200 & \text{if } t > 10.
\end{cases}
\]

Now we solve for \( t \) in \( s(t) = 1000 \).

\[
30t - 200 = 1000 \\
30t = 1200 \\
t = 40
\]

So it will take 40 seconds for the car to travel 1 km.

V. A car is traveling at 60 mi/h when the brakes are fully applied, producing a constant deceleration of 15 ft/s^2. What is the distance traveled before the car comes to a stop?

\[
\frac{60 \text{ mi}}{h} = \frac{60 \text{ mi}}{h} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ h}}{3600 \text{ s}} = \frac{5280 \text{ ft}}{60 \text{ s}} = \frac{264 \text{ ft}}{3 \text{ s}} = \frac{88 \text{ ft}}{s}
\]

The velocity and position functions for the car are:

\[
v(t) = -15t + 88 \\
s(t) = -15t^2/2 + 88t
\]

So we need to solve

\[
v(t) = -15t + 88 = 0 \implies t = \frac{88}{15}
\]

It will take \( t = \frac{88}{15} \) seconds for the car to stop. The car will travel \( s(\frac{88}{15}) = \frac{-15 \cdot 88^2}{2 \cdot 15^2} + 88 \cdot \frac{88}{15} = \frac{-88^3}{15} + 88 \cdot \frac{88}{15} = \frac{3872}{15} \) feet.

VI. The speed of a runner increased steadily during 3 seconds of a race. Her speeds are given below. Find lower and upper estimates for the distance that she traveled during these three seconds.
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The lower estimate:

\[ LE = \frac{1}{2} v(0.5) + \frac{1}{2} v(1) + \frac{1}{2} v(1.5) + \frac{1}{2} v(2) + \frac{1}{2} v(2.5) + \frac{1}{2} v(3) \]

\[ = \frac{1}{2} (v(0.5) + v(1) + v(1.5) + v(2) + v(2.5) + v(3)) \]

\[ = \frac{1}{2} (18 + 14 + 10 + 8 + 4 + 0) \]

\[ = \frac{1}{2} (54) \]

\[ = 27 \]

The upper estimate:

\[ UE = \frac{1}{2} v(0) + \frac{1}{2} v(0.5) + \frac{1}{2} v(1) + \frac{1}{2} v(1.5) + \frac{1}{2} v(2) + \frac{1}{2} v(2.5) \]

\[ = \frac{1}{2} (v(0) + v(0.5) + v(1) + v(1.5) + v(2) + v(2.5)) \]

\[ = \frac{1}{2} (20 + 18 + 14 + 10 + 8 + 4) \]

\[ = \frac{1}{2} (74) \]

\[ = 37 \]

The runner traveled at least 27 feet but less than 37 feet.

VII. Use the form of the definition of the integral given in Theorem 4 to evaluate the integral.

1.

\[ \int_{0}^{3} 5dx \]
Answer:

\[ \int_0^3 5\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i)\Delta x \]
\[ = \lim_{n \to \infty} \sum_{i=1}^{n} f(0 + \frac{3}{n}i) \frac{3}{n} \]
\[ = \lim_{n \to \infty} \sum_{i=1}^{n} \frac{3}{n} \]
\[ = \lim_{n \to \infty} \sum_{i=1}^{n} \frac{15}{n} \]
\[ = \lim_{n \to \infty} \frac{15}{n} \sum_{i=1}^{n} 1 \]
\[ = \lim_{n \to \infty} \frac{15}{n} \]
\[ = \lim_{n \to \infty} 15 \]
\[ = 15 \]

2.

\[ \int_0^2 (3 - 2x - 3x^2)\,dx \]
Answer:

\[
\int_{0}^{2} (3 - 2x - 3x^2) dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x \\
= \lim_{n \to \infty} \sum_{i=1}^{n} f(0 + \frac{2i}{n}) \frac{2}{n} \\
= \lim_{n \to \infty} \sum_{i=1}^{n} (3 - 2(\frac{2i}{n}) - 3(\frac{2i}{n})^2) \frac{2}{n} \\
= \lim_{n \to \infty} \sum_{i=1}^{n} \left( \frac{6}{n} - i \frac{8}{n^2} - i^2 \frac{24}{n^3} \right) \\
= \lim_{n \to \infty} \sum_{i=1}^{n} \frac{6}{n} - \sum_{i=1}^{n} \frac{i}{n^2} - \sum_{i=1}^{n} \frac{i^2}{n^3} \\
= \lim_{n \to \infty} \frac{6}{n} \sum_{i=1}^{n} 1 - \frac{8}{n^2} \sum_{i=1}^{n} i - \frac{24}{n^3} \sum_{i=1}^{n} i^2 \\
= \lim_{n \to \infty} \frac{6}{n} - \frac{8}{n^2} \frac{n(n+1)}{2} - \frac{24}{n^3} \frac{n(n+1)(2n+1)}{6} \\
= 6 - 4 - 8 \\
= -6
\]

3.

\[
\int_{a}^{b} 5 dx 
\]
Answer:

\[
\int_a^b 5dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x
\]

\[
= \lim_{n \to \infty} \sum_{i=1}^{n} f(a + i \frac{b-a}{n}) \frac{b-a}{n}
\]

\[
= \lim_{n \to \infty} \sum_{i=1}^{n} \frac{5(b-a)}{n}
\]

\[
= \lim_{n \to \infty} 5 \frac{b-a}{n} \sum_{i=1}^{n} 1
\]

\[
= \lim_{n \to \infty} 5 \frac{b-a}{n} n
\]

\[
= \lim_{n \to \infty} 5(b-a)
\]

\[
= 5(b-a)
\]

4.

\[
\int_a^b xdx
\]
Answer:

\[ \int_{a}^{b} x \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x \]

\[ = \lim_{n \to \infty} \sum_{i=1}^{n} f \left( a + i \frac{b-a}{n} \right) \frac{b-a}{n} \]

\[ = \lim_{n \to \infty} \sum_{i=1}^{n} \left( a + i \frac{b-a}{n} \right) \frac{b-a}{n} \]

\[ = \lim_{n \to \infty} \sum_{i=1}^{n} \frac{a(b-a)}{n} + i \frac{(b-a)^2}{n} \]

\[ = \lim_{n \to \infty} \frac{a(b-a)}{n} \sum_{i=1}^{n} 1 + \frac{(b-a)^2}{n} \sum_{i=1}^{n} i \]

\[ = \lim_{n \to \infty} \frac{a(b-a)}{n} n + \frac{(b-a)^2}{n} \frac{n(n+1)}{2} \]

\[ = a(b-a) + \frac{(b-a)^2}{2} \]

\[ = ab - a^2 + \frac{b^2}{2} - \frac{2ab}{2} + \frac{a^2}{2} \]

\[ = \frac{b^2}{2} - \frac{a^2}{2} \]

5.

\[ \int_{a}^{b} x^2 \, dx \]

Answer:
\[ \int_a^b x^2 \, dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x \]

\[ = \lim_{n \to \infty} \sum_{i=1}^{n} f(a + i \frac{b-a}{n}) \frac{b-a}{n} \]

\[ = \lim_{n \to \infty} \sum_{i=1}^{n} (a + i \frac{b-a}{n})^2 \frac{b-a}{n} \]

\[ = \lim_{n \to \infty} \sum_{i=1}^{n} (a^2 + i2 \frac{a(b-a)}{n} + i^2 \frac{(b-a)^2}{n^2}) \frac{b-a}{n} \]

\[ = \lim_{n \to \infty} \sum_{i=1}^{n} (a^2 \frac{b-a}{n} + i2 \frac{a(b-a)^2}{n^2} + i^2 \frac{(b-a)^3}{n^3}) \]

\[ = \lim_{n \to \infty} \left( a^2 \frac{b-a}{n} n + 2 \frac{a(b-a)^2}{n^2} \frac{n(n+1)}{2} + \frac{(b-a)^3}{n^3} n(n+1)(2n+1) \right) \]

\[ = a^2 (b-a) + a(b-a)^2 + \frac{(b-a)^3}{3} \]

\[ = \frac{b^3}{3} - \frac{a^3}{3} \]

6.

\[ \int_a^b x^3 \, dx \]

Answer:
\[
\int_a^b x^3 dx = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x
\]
\[
= \lim_{n \to \infty} \sum_{i=1}^{n} f\left(a + i \frac{b-a}{n}\right) \frac{b-a}{n}
\]
\[
= \lim_{n \to \infty} \sum_{i=1}^{n} \left(a + i \frac{b-a}{n}\right)^3 \frac{b-a}{n}
\]
\[
= \lim_{n \to \infty} \sum_{i=1}^{n} \left(a^3 + 3a^2 \frac{b-a}{n} \frac{(b-a)^2}{2} + 3a \frac{(b-a)^3}{3} \frac{b-a}{n} \right)
\]
\[
= \lim_{n \to \infty} \left(a^3 \frac{b-a}{n} + 3a^2 \frac{(b-a)^2}{2} + 3a \frac{(b-a)^3}{3} + \frac{1}{n} (b-a)^4\right)
\]
\[
= \frac{b^4}{4} - \frac{a^4}{4}
\]

7. \[
\int_a^b (cx^3 + dx^2 + ex + f) dx
\]

Answer:

\[
\int_a^b (cx^3 + dx^2 + ex + f) dx = c\int_a^b x^3 dx + d\int_a^b x^2 dx + e\int_a^b x dx + f\int_a^b dx
\]
\[
= c\left(\frac{b^4}{4} - \frac{a^4}{4}\right) + d\left(\frac{b^3}{3} - \frac{a^3}{3}\right) + e\left(\frac{b^2}{2} - \frac{a^2}{2}\right) + f(b-a)
\]

VIII. Evaluate the following integrals

1. \[
\int_{-\pi/2}^{\pi/2} \sin(x) + \cos(2x) dx
\]
Answer:

\[ \int_{-\pi/2}^{\pi/2} \sin(x) + \cos(2x) \, dx = \int_{-\pi/2}^{\pi/2} \sin(x) \, dx + \int_{-\pi/2}^{\pi/2} \cos(2x) \, dx \]
\[ = 0 + 0 \]
\[ = 0 \]

2.

\[ \int_{-2}^{0} (2 + \sqrt{4 - x^2}) + \sqrt{4 - (x + 2)^2} \, dx \]

Answer:

\[ \int_{-2}^{0} (2 + \sqrt{4 - x^2}) + \sqrt{4 - (x + 2)^2} \, dx = \int_{-2}^{0} (2 + \sqrt{4 - x^2}) \, dx + \int_{-2}^{0} \sqrt{4 - (x + 2)^2} \, dx \]
\[ = 2 \cdot 2 + \frac{1}{4} \pi 2^2 + \frac{1}{4} \pi 2^2 \]
\[ = 4 + \pi + \pi \]
\[ = 4 + 2\pi \]

3.

\[ \int_{-1}^{2} |x + 1| \, dx \]

Answer:

\[ \int_{-1}^{2} |x + 1| \, dx = \int_{-1}^{2} x + 1 \, dx \]
\[ = \frac{1}{2} 3 \cdot 3 \]
\[ = \frac{9}{2} \]

4.

\[ \int_{-1}^{2} |x + 1| - 2|x - 1| \, dx \]
Answer:

\[
\int_{-1}^{2} |x + 1| - 2|x - 1| \, dx = \int_{-1}^{2} |x + 1| \, dx - 2 \int_{-1}^{2} |x - 1| \, dx
\]

\[
= \int_{-1}^{2} (x + 1) \, dx + 2 \int_{-1}^{1} (x - 1) \, dx - 2 \int_{1}^{2} (x - 1) \, dx
\]

\[
= \frac{9}{2} + 2 \cdot \frac{2}{2} - 2 \cdot \frac{1}{2} \cdot 1
\]

\[
= \frac{9}{2} + 4 - 1
\]

\[
= \frac{9}{2} + 3
\]

\[
= \frac{15}{2}
\]

5.

Answer:

\[
\int_{0}^{2} |x^2 + 1| \, dx
\]

\[
\int_{0}^{2} |x^2 + 1| \, dx = \int_{0}^{2} x^2 + 1 \, dx
\]

\[
= \int_{0}^{2} x^2 \, dx + \int_{0}^{2} 1 \, dx
\]

\[
= \frac{2^3}{3} - \frac{0^3}{3} + 1(2 - 0)
\]

\[
= \frac{8}{3} + 2
\]

\[
= \frac{14}{3}
\]