Name ____________________________

(circle your TA discussion section)

- AD1 - 9:00 Mary Angelica  
- ADB - 9:00 Adriana  
- ADE - 12:00 Hsin-Po  
- ADH - 3:00 Ravi  
- ADO - 2:00 Chaeryn  
- AD2 - 1:00 Stefan  
- ADC - 10:00 Xunjun (Henry)  
- ADF - 1:00 Artur  
- ADJ - 9:00 Ciaran  
- ADM - 12:00 Dara  
- ADQ - 4:00 Chaeryn  
- ADA - 8:00 Maria  
- ADD - 11:00 Artur  
- ADG - 2:00 Maria  
- ADK - 10:00 Ciaran  
- ADN - 1:00 Hsin-Po  
- ADR - 10:00 Adriana

- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else’s work.
- You may use your notes, the textbook, or information found on my course home page.
- You may use a calculator only for basic arithmetic. In particular you should not use its graphing features.
- You are not allowed to search the Internet, use Wolfram Alpha, or use technology for anything beyond what is stated above.
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- The quiz is due at the beginning of your official discussion period on Tuesday, April 3rd
- Note to TAs and Tutors – you should not help students with these specific problems until all discussion sections have turned in the quiz.
1. (2 points) Find a formula for the function $f(\theta)$ given that $f''(\theta) = 6\cos(\theta) - 4\sin(\theta)$, $f(0) = 5$ and $f(\pi) = 10$.

\[ f''(\theta) = 6\cos(\theta) - 4\sin(\theta) \]

\[ f'(\theta) = 6\sin(\theta) + 4\cos(\theta) + C_1 \]

\[ f(\theta) = -6\cos(\theta) + 4\sin(\theta) + C_1 \theta + C_2 \]

\[ f(0) = -6 \cdot 1 + 4 \cdot 0 + C_1 \cdot 0 + C_2 = 5 \]

\[ \Rightarrow -6 + C_2 = 5 \]

\[ \Rightarrow C_2 = 11 \]

\[ f(\theta) = -6\cos(\theta) + 4\sin(\theta) + C_1 \theta + 11 \]

\[ f(\pi) = 10 \]

\[ -6\cos(\pi) + 4\sin(\pi) + C_1 \pi + 11 = 10 \]

\[ 6 + 0 + \pi \cdot C_1 + 11 = 10 \]

\[ \pi \cdot C_1 = 10 - 11 - 6 \]

\[ \pi \cdot C_1 = -7 \]

\[ C_1 = -\frac{7}{\pi} \]

\[ f(\theta) = -6\cos(\theta) + 4\sin(\theta) - \frac{7}{\pi} \theta + 11 \]
2. (2 points) Suppose that \( r(x) \) is continuous at all real numbers and satisfies the following equations.

\[
\begin{align*}
&\int_{2}^{7} r(x) \, dx = 7 \\
&\int_{2}^{5} r(x) \, dx = 8 \\
&\int_{2}^{3} r(x) \, dx = 9
\end{align*}
\]

What is the value of \( \int_{5}^{7} (2r(x) + 1) \, dx \)?

\[
\begin{align*}
\int_{5}^{7} (2r(x) + 1) \, dx &= \int_{5}^{7} (2r(x) + 1) \, dx - \int_{2}^{5} (2r(x) + 1) \, dx - \int_{2}^{3} (2r(x) + 1) \, dx \\
&= 2 \int_{2}^{7} r(x) \, dx + \int_{2}^{7} 1 \, dx - 2 \int_{3}^{5} r(x) \, dx - 2 \int_{2}^{5} 1 \, dx - \int_{2}^{3} r(x) \, dx - \int_{2}^{3} 1 \, dx \\
&= 2 \times (7) + x \int_{2}^{7} - 2 \times (-8) - x \int_{3}^{5} - 2 \times (9) - x \int_{2}^{3} \\
&= 14 + \left( \left[ (7) - (2) \right] +16 - \left[ (5) - (3) \right] \right) - 18 - \left[ (3) - (2) \right] \\
&= 14 + 5 + 16 - 2 - 18 - 1 \\
&= 14
\end{align*}
\]
3. (2 points) Evaluate the following limit. Use proper notation in each step.

\[
\lim_{n \to \infty} \sum_{k=1}^{n} \frac{3n^2 + 4kn - 12k^2 - 5}{n^3}
\]

\[
= \lim_{n \to \infty} \left( \sum_{k=1}^{n} \frac{3}{n} + \frac{4k}{n^2} - \frac{12k^2}{n^3} - \frac{5}{n^3} \right)
\]

\[
= \lim_{n \to \infty} \left[ \frac{3}{n} \sum_{k=1}^{n} 1 + \frac{4}{n^2} \sum_{k=1}^{n} k - \frac{12}{n^3} \sum_{k=1}^{n} k^2 - \frac{5}{n^3} \right]
\]

\[
= \lim_{n \to \infty} \left[ \frac{3}{n} \frac{n(n+1)}{2} + \frac{4}{n^2} \frac{n(n+1)}{2} - \frac{12}{n^3} \frac{n(n+1)(2n+1)}{6} - \frac{5}{n^3} \right]
\]

\[
= \lim_{n \to \infty} \left[ 3 + \frac{2n^2 + 2n}{n^2} - \frac{4n^3 + 6n^3 + 2n}{n^3} - \frac{5}{n^3} \right]
\]

\[
= \lim_{n \to \infty} \left[ 3 + \frac{2 + \frac{2}{n}}{1} - \frac{4 + \frac{6}{n^2} + \frac{2}{n}}{1} - \frac{5}{n^2} \right]
\]

\[
= 3 + 2 - 4 - 0
\]

\[
= 1
\]
4. (2 points) From section 5.2 we have the following property of definite integrals.

\[
\text{If } f(x) \text{ is continuous and } m \leq f(x) \leq M \text{ for } a \leq x \leq b, \text{ then } m(b-a) \leq \int_{a}^{b} f(x) \, dx \leq M(b-a)
\]

Use this property to carefully explain why the following inequality holds.

\[
3.2 \leq \int_{2}^{10} \frac{1}{\frac{1}{2} \cos^{6}(x^3) + \sin^{8}(x^8) + 1} \, dx \leq 8
\]

\[
-1 \leq \cos(x) \leq 1, \quad -1 \leq \sin(x) \leq 1
\]

\[
0 \leq \cos^{6}(x^3) \leq 1, \quad 0 \leq \sin^{8}(x^8) \leq 1
\]

\[
0 \leq \frac{1}{2} \cos^{6}(x^3) + \sin^{8}(x^8) + 1 \leq \frac{1}{2} + 1 + 1 \leq \frac{5}{2}
\]

\[
1 \geq \frac{1}{\frac{1}{2} \cos^{6}(x^3) + \sin^{8}(x^8) + 1} \geq \frac{2}{5}
\]

\[
\frac{2}{5} \leq f(x) \leq 5
\]

Since \( f(x) \) is continuous, we have

\[
\frac{2}{5} \left( 10 - 2 \right) \leq \int_{2}^{10} f(x) \, dx \leq 5 \left( 10 - 2 \right)
\]

\[
\frac{16}{5} = 3.2
\]
5. (2 points) At time $t$ seconds, the velocity of an object is $v(t) = t^2 + e^{2t}$ m/s. The distance in meters traveled by this object from $t = 3$ to $t = 7$ can be written as a limit of Riemann sums in many different ways. I have shown how to do this for two of the six ways indicated below. Fill in the missing information for the remaining limits so that the only variables appearing are $n$ and $k$. Do not evaluate these limits.

(a) Using a limit of midpoint Riemann sums,

$$
DISTANCE = \lim_{n \to \infty} \sum_{k=1}^{n} \left[ \left( 3 + \left( k - 0.5 \right) \cdot \frac{4}{n} \right)^2 + e^{2 \left( 3 + \left( k - 0.5 \right) \cdot \frac{4}{n} \right)} \right] \cdot \frac{4}{n}
$$

(b) Using a limit of left Riemann sums,

$$
DISTANCE = \lim_{n \to \infty} \sum_{k=1}^{n} \left[ \left( 3 + \left( k-1 \right) \cdot \frac{4}{n} \right)^2 + e^{2 \left( 3 + \left( k-1 \right) \cdot \frac{4}{n} \right)} \right] \cdot \frac{4}{n}
$$

(c) Using a limit of right Riemann sums,

$$
DISTANCE = \lim_{n \to \infty} \sum_{k=1}^{n} \left[ \left( 3 + k \cdot \frac{4}{n} \right)^2 + e^{2 \left( 3 + k \cdot \frac{4}{n} \right)} \right] \cdot \frac{4}{n}
$$

(d) Using a limit of midpoint Riemann sums,

$$
DISTANCE = \lim_{n \to \infty} \sum_{k=0}^{n-1} \left[ \left( 3 + \left( k + \frac{1}{2} \right) \cdot \frac{4}{n} \right)^2 + e^{2 \left( 3 + \left( k + \frac{1}{2} \right) \cdot \frac{4}{n} \right)} \right] \cdot \frac{4}{n}
$$

(e) Using a limit of left Riemann sums,

$$
DISTANCE = \lim_{n \to \infty} \sum_{k=0}^{n-1} \left[ \left( 3 + k \cdot \frac{4}{n} \right)^2 + e^{2 \left( 3 + k \cdot \frac{4}{n} \right)} \right] \cdot \frac{4}{n}
$$

(f) Using a limit of right Riemann sums,

$$
DISTANCE = \lim_{n \to \infty} \sum_{k=0}^{n-1} \left[ \left( 3 + \left( k + 1 \right) \cdot \frac{4}{n} \right)^2 + e^{2 \left( 3 + \left( k + 1 \right) \cdot \frac{4}{n} \right)} \right] \cdot \frac{4}{n}
$$