Solutions

Circle your TA discussion section.

- AD1 - 9:00 Mary Angelica
- ADB - 9:00 Adriana
- ADE - 12:00 Hsin-Po
- ADH - 3:00 Ravi
- ADL - 11:00 Dara
- ADO - 2:00 Chaeryn
- AD2 - 1:00 Stefan
- ADC - 10:00 Xunjun (Henry)
- ADF - 1:00 Artur
- ADJ - 9:00 Ciaran
- ADM - 12:00 Dara
- ADQ - 4:00 Chaeryn
- ADA - 8:00 Maria
- ADD - 11:00 Artur
- ADG - 2:00 Maria
- ADK - 10:00 Ciaran
- ADN - 1:00 Hsin-Po
- ADR - 10:00 Adriana

- Sit in your assigned seat (circled below).
- Do not open this test booklet until I say START.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- There is no partial credit on multiple-choice questions. For all other questions, you must show sufficient work to justify your answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say STOP.
- Quickly turn in your test to me or a TA and show your Student ID.

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FRONT OF ROOM - 141 Wohlers Hall
1. (10 points) Find \( g'(q) \) given that \( g(q) = \left( \frac{\ln(q^7)}{\arctan(2q)} \right)^g \)

\[
\begin{align*}
g'(q) &= q \left( \frac{\ln(q^7)}{\arctan(2q)} \right)^g \cdot \left[ \frac{\arctan(2q)}{q^2}, \frac{7q^6}{1 + (2q)^2}, -\ln(q^7), \frac{1}{(\arctan(2q))^2} \right]
\end{align*}
\]

2. (10 points) Find \( s'(x) \) given that \( s(x) = (32 - x)^{\frac{1}{x^2}} \)

\[
\begin{align*}
y &= (32 - x)^{\frac{1}{x^2}} \\
\ln(y) &= \ln((32 - x)^{\frac{1}{x^2}}) \\
\left( \ln(y) \right)' &= \left( \frac{1}{x^2}, \ln((32 - x)) \right)' \\
\frac{1}{y} \cdot y' &= \left[ x^{-2} \left( \frac{1}{32 - x} \right) + \ln(32 - x) \left( -2x^{-3} \right) \right] \\
\frac{1}{y} \cdot y' &= (32 - x)^{\frac{1}{x^2}} \left[ \frac{-1}{x^2(32 - x)} - \frac{2\ln(32 - x)}{x^3} \right]
\end{align*}
\]
3. (10 points) Find \( h'(s) \) given that \( h(s) = \cos^4(2s)e^{s^2-3s} \)

\[
h'(s) = \cos^4(2s) \left[ e^{s^2-3s} (2s-3) \right] + e^{s^2-3s} \left[ 4 \cos^3(2s)(-\sin(2s)) \right] (2)
\]

4. (10 points) Find \( \frac{dy}{dx} \) given that \( \frac{x^5}{\sin(y)} = 7x^5 - 4y^2 \)

\[
\left( \frac{x^5}{\sin(y)} \right)' = (7x^5 - 4y^2)' \\
\sin(y) (5x^4) - x^5 \cos(y) y' = 35x^4 \sin^2(y) - 8y y' \sin^2(y)
\]

\[
\sin(y) (5x^4) - x^5 \cos(y) y' = 35x^4 \sin^2(y) - 8y y' \sin^2(y)
\]

\[
y' \left[ 8y \sin^2(y) - x^5 \cos(y) \right] = 35x^4 \sin^2(y) - \sin(y) 5x^4
\]

\[
\frac{dy}{dx} = \frac{35x^4 \sin^2(y) - \sin(y) 5x^4}{8y \sin^2(y) - x^5 \cos(y)}
\]
5. (10 points) Solve the following differential equations given that the graph of each solution goes through the point \((\theta, h) = (0, 5)\). You must use the given variables.

(a) \[
\frac{dh}{d\theta} = 16\theta
\]

\[
h = 8\theta^2 + C
\]

\((0, 5)\) \[
5 = 8(0)^2 + C
\]

\[
h = 8\theta^2 + 5
\]

(b) \[
\frac{dh}{d\theta} = 16h
\]

\[
h = Ce^{16\theta}
\]

\((0, 5)\) \[
5 = Ce^{16(0)}
\]

\[
h = 5e^{16\theta}
\]

6. (10 points) Evaluate the limit. You must fully justify your answer.

\[
\lim_{x \to 0} \frac{\sin(7x) - 7e^x + 7}{e^{2x} - 6x - 1}
\]

\[
= \lim_{x \to 0} \frac{7\cos(7x) - 7e^x}{6e^x - 6}
\]

\[
= \lim_{x \to 0} \frac{-49\sin(7x) - 7e^x}{36e^x}
\]

\[
= -\frac{49\sin(0) - 7e^0}{36e^0} = -\frac{7}{36}
\]
7. (10 points) Let \( f(x) = \frac{4x}{\ln(x)} \). Determine each interval where \( f \) is increasing and each interval where \( f \) is decreasing.

\[
\frac{d}{dx} f(x) = \frac{\ln(x)\left(\frac{4}{x}\right) - 4x\left(\frac{1}{x}\right)}{(\ln(x))^2}
\]

**Domain** \( x > 0 \) and \( \ln(x) \neq 0 \)

Critical Points: \( f'(x) = 0 \) or undefined

**Critical Values**

\( 4\ln(x) - 4 = 0 \)
\( 4\ln(x) = 4 \)
\( \ln(x) = 1 \)
\( x = e \)

\( f'(e^{\frac{1}{2}}) = \frac{4\ln(e^{\frac{1}{2}}) - 4}{(\ln(e^{\frac{1}{2}}))^2} = - \)

\( f'(e^{\frac{3}{2}}) = \frac{4\ln(e^{\frac{3}{2}}) - 4}{(\ln(e^{\frac{3}{2}}))^2} = \frac{2\ln(e) - 4}{(\ln(e^{\frac{3}{2}}))^2} = - \)

\( f'(e^2) = \frac{4\ln(e^2) - 4}{(\ln(e^2))^2} = \frac{8\ln(e) - 4}{(\ln(e^2))^2} = + \)

\( f(x) \) is increasing on \((e, \infty)\)
\( f(x) \) is decreasing on \((0, 1)\) and \((1, e)\)
8. (10 points) A function $f(x)$ is continuous at each real number and it has the following first derivative (below). Find each interval where $f(x)$ is concave up, each interval where $f(x)$ is concave down, and list the x-coordinate of each inflection point.

$$f'(x) = 3x^4 - 32x^3 + 96x^2$$

$$f''(x) = 12x^3 - 96x^2 + 192x$$

$$f'''(x) = 0 \quad (\text{or undefined})$$

$$12x(x^2 - 8x + 16) = 0$$

$$12x(x - 4)^2 = 0$$

$$x = 0, 4$$

\[\begin{array}{cccc}
- & 0 & + & + \\
\downarrow & x & x & x \\
0 & 4 \\
\end{array}\]

$$f'''(-1) = 12(-1)(-1-4)^2 = -$$

$$f'''(2) = 12(2)(2-4)^2 = +$$

$$f'''(5) = 12(5)(5-4)^2 = +$$

$f(x)$ is concave down on $(-\infty, 0)$

$f(x)$ is concave up on $(0, 4)$ and $(4, \infty)$

$x = 0$ is an inflection point
9. (10 points) Find the x-coordinate of the point on the graph of \( y = 2x + 2 \) which is closest to the point \((3,0)\).

\[
d = \sqrt{(x-3)^2 + (2x+2-0)^2}
\]

Want: Minimize distance from \((x,2x+2)\) to \((3,0)\).

The x-coordinate which minimizes \(d\) also minimizes \(d^2\).

\[
d^2 = x^2 - 6x + 9 + 4x^2 + 8x + 4
\]

\[
d^2 = 5x^2 + 2x + 13
\]

\[
\left(\frac{d^2}{dx}\right) = 10x + 2
\]

\[
10x + 2 = 0
\]

\[
x = -\frac{1}{5}
\]

\[
f'(-1) = -
\]

\[
f'(0) = +
\]
10. (10 points) A 12 foot ladder rests against a wall, making an angle $\theta$ between the ladder and the ground. The ladder begins to slide down the wall. When the bottom of the ladder is 4 feet from the wall, the angle $\theta$ is decreasing at a rate of 0.5 radians/sec. How fast is the ladder sliding down the wall at this time?

\[
\sin \theta = \frac{y}{12}
\]

\[
(\sin \theta)' = \left( \frac{1}{12} y \right)'
\]

\[
(\cos \theta) \cdot \frac{d\theta}{dt} = \frac{1}{12} \cdot \frac{dy}{dt}
\]

\[
\frac{dy}{dt} = 12 \cos \theta \cdot \frac{d\theta}{dt}
\]

\[
= 12 \cos \theta \left( -0.5 \text{ radians/sec} \right)
\]

\[
= -6 \cos \theta
\]

\[
= -6 \left( \frac{4}{12} \right) = -6 \left( \frac{1}{3} \right) = -2 \text{ ft/sec}
\]
Students – do not write on this page!

1. (10 points) ________________________________

2. (10 points) ________________________________

3. (10 points) ________________________________

4. (10 points) ________________________________

5. (10 points) ________________________________

6. (10 points) ________________________________

7a. (10 points) ________________________________

8. (10 points) ________________________________

9. (10 points) ________________________________

10. (10 points) ________________________________

TOTAL (100 points) ________________