Circle your TA discussion section.

- AD1 - 9:00 Mary Angelica
- AD2 - 1:00 Stefan
- ADB - 9:00 Adriana
- ADC - 10:00 Xunjun (Henry)
- ADE - 12:00 Hsin-Po
- ADH - 3:00 Ravi
- ADJ - 9:00 Ciaran
- ADL - 11:00 Dara
- ADM - 12:00 Dara
- ADO - 2:00 Chaeryn
- ADQ - 10:00 Chaeryn
- ADA - 8:00 Maria
- ADD - 11:00 Artur
- ADG - 2:00 Maria
- ADK - 10:00 Ciaran
- ADN - 1:00 Hsin-Po
- ADR - 10:00 Adriana

- Sit in your assigned seat (circled below).
- Do not open this test booklet until I say START.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- Remove hats and sunglasses.
- There is no partial credit on multiple-choice questions. For all other questions, you must show sufficient work to justify your answer.
- While the test is in progress, we will not answer questions concerning the test material.
- Do not leave early unless you are at the end of a row.
- Quit working and close this test booklet when I say STOP.
- Quickly turn in your test to me and a TA and show your Student ID.

FRONT OF ROOM - 141 Wohlers Hall
1. (20 points) Circle **True** if the statement is always true. Otherwise circle **false**.

(a) (5 points) If \( \lim_{x \to a} f(x) \) exists, then \( f(x) \) is continuous at \( x = a \).

true or false?

(b) (5 points) The function \( f(x) = \frac{4 - x}{x^2 - 7x + 12} \) has a vertical asymptote at \( x = 4 \).

true or false?

\[
\begin{align*}
\frac{4 - x}{(x-4)(x-3)} &= \frac{-1}{x-3} \quad f(x) \text{ has a hole at } x=4
\end{align*}
\]

(c) (5 points) Let \( f(x) \) be a continuous function on the closed interval \([1,10]\). If \( f(2) = -5 \) and \( f(5) = 5 \), then it must be true that there is some number \( c \), \( 2 < c < 5 \), such that \( f(c) = 3 \).

true or false?

Intermediate Value Theorem

(d) (5 points) If \( v(t) \) is an odd function, and \( w(t) \) is an odd function, then \( p(t) = (v(w(t)))^2 \) is an odd function.

true or false?

\[
\begin{align*}
p(-t) &= (v(w(-t)))^2 \quad \text{is odd} \\
      &= (v(-w(t)))^2 \quad \text{is odd} \\
      &= (-v(w(t)))^2 \quad v(t) \text{ is odd} \\
      &= (v(w(t)))^2 \\
      &= p(t) \quad \text{is even}
\end{align*}
\]
2. (10 points) Find the equation of each horizontal asymptote for the following graph. Your answer must be justified using limits.

\[
\lim_{x \to \infty} \frac{7 + 8x^3}{x^3 - 4} = \lim_{x \to \infty} \frac{7/x^3 + 8/x^3}{x^3/x^3} = \frac{8}{1} = 8
\]

\[
\lim_{x \to -\infty} \frac{7 + 8x^3}{x^3 - 4} = \lim_{x \to -\infty} \frac{7/x^3 + 8/x^3}{x^3/x^3} = \frac{8}{1} = 8
\]

\[
\text{HA at } y = 8
\]

3. (10 points) Determine a formula for an exponential function, given that its graph goes through the points \((0, e^6)\) and \((4, e^{20})\).

\[
y = C \cdot a^x
\]

\[
(0, e^6) \quad (4, e^{20})
\]

\[
e^6 = C \cdot a^0
\]

\[
e^6 = C
\]

\[
e^6 = C \cdot a^4
\]

\[
e^{20} = e^6 \cdot a^4
\]

\[
e^{20}/e^6 = a^4
\]

\[
e^{16} = a^4
\]

\[
a = \sqrt[4]{e^{16}}
\]

\[
y = e^{6 \cdot (\sqrt[4]{e^{16}})^x}
\]
4. (10 points) Let \( f(x) = 2x^2 - 3x \).

Use the definition of a derivative as a limit to prove that \( f'(x) = 4x - 3 \).

Show each step in your calculation and be sure to use proper terminology in each step of your proof.

\[
\begin{align*}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{(2(x+h)^2 - 3(x+h)) - (2x^2 - 3x)}{h} \\
&= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 3x - 3h - 2x^2 + 3x}{h} \\
&= \lim_{h \to 0} \frac{2x^2 + 4xh + 2h^2 - 3h - 2x^2}{h} \\
&= \lim_{h \to 0} \frac{h(4x + 2h - 3)}{h} \\
&= \lim_{h \to 0} (4x + 2h - 3) = 4x + 2(0) = 3 \\
&= 4x - 3
\end{align*}
\]
5. (10 points) The function \( q(t) = \frac{\ln(7)}{4e^{6t}} \) is one-to-one on its domain. Determine a formula for its inverse \( q^{-1}(t) \).

\[
q = \frac{\ln(7)}{4e^{6q}}
\]

\[
t = \frac{\ln(7)}{4e^{6q}}
\]

\[
4e^{6q}t = \ln(7)
\]

\[
e^{6q} = \frac{\ln(7)}{4t}
\]

\[
\ln(e^{6q}) = \ln\left(\frac{\ln(7)}{4t}\right)
\]

\[
\frac{1}{6} \cdot \ln(e) = \ln\left(\frac{\ln(7)}{4t}\right)
\]

\[
q = \frac{1}{6} \ln\left(\frac{\ln(7)}{4t}\right)
\]

\[
q^{-1}(t) = \frac{1}{6} \ln\left(\frac{\ln(7)}{4t}\right)
\]

6. (12 points) Determine the domain of the following function.

\[
g(x) = \frac{\ln(2 - 3x) + \sin(3e^{4x})}{1 - e^{5x}}
\]

\[
2 - 3x > 0
\]

\[
2 > 3x
\]

\[
\frac{2}{3} > x
\]

\[
\text{Domain: } (-\infty, 0) \cup (0, \frac{2}{3})
\]
7. (6 points each) Evaluate the following limits without the use of derivatives. Show sufficient justification for each answer. An answer of ‘does not exist’ is not sufficient. For infinite limits you must state if it is \( \infty \) or \( -\infty \).

(a) \( \lim_{x \to 8} \frac{16 - 2x}{4 - \sqrt{2x}} = \lim_{x \to 8} \frac{16 - 2x}{4 - \sqrt{2x}} \cdot \frac{\sqrt{2x}}{\sqrt{2x}} = \lim_{x \to 8} \frac{16 - 2x}{4 - 2x} \cdot \frac{\sqrt{2x}}{\sqrt{2x}} = \lim_{x \to 8} \frac{16 - 2x}{4 - 2x} = \frac{4 + \sqrt{2} \cdot 8}{4 + \sqrt{2}} = 4 + 4 = 8 \)

(b) \( \lim_{x \to 3} \frac{x - 4}{\ln(4 - x)} \)
(c) \[ \lim_{x \to \infty} \frac{\sin(x^2 \cos(x))}{e^x} \to +\infty \]

So by the Squeeze Theorem

\[ \lim_{x \to \infty} \frac{\sin(x^2 \cos(x))}{e^x} = 0 \]

(d) \[ \lim_{x \to \pi/4} \frac{\cos(2x)}{\cos(x) - \sin(x)} \]

\[
\begin{align*}
\cos(\pi/4) &= \cos(\pi/4) = \frac{\sqrt{2}}{2} \\
\sin(\pi/4) &= \sin(\pi/4) = \frac{\sqrt{2}}{2} \\
\end{align*}
\]

\[ = \frac{\cos(x) - \sin(x)}{(\cos(x) + \sin(x))} \]

\[ = \lim_{x \to \pi/4} \frac{\cos(x) + \sin(x)}{\cos(x) + \sin(x)} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2} \]

(e) \[ \lim_{x \to 1^+} \frac{e^{2x}}{\sin\left(\frac{x-1}{1-x} \right)} \]

\[ \frac{e^2}{0} = -\infty \]

\[ x = 1.1 \]

\[ \sin\left(\frac{1.1-1}{1.1-1} \right) = \sin\left(\frac{1}{0} \right) = \sin\left(-\frac{1}{0} \right) \]

\[ \text{negative number} \]