Definition of the Derivative

\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \] (provided this finite limit exists)

The term **differentiation** refers to the process of finding a derivative.

Derivative Notation

Given a function \( y = f(x) \), we have the following ways to denote its derivative with respect to \( x \).

- \( f'(x) \)
- \( y' \)
- \( \frac{d}{dx} (f(x)) \)
- \( \frac{d}{dx} (y) = \frac{dy}{dx} \) (Leibniz notation)

Interpretation of the Derivative

At a specific point, the following represent the same quantity.

- The derivative
- The slope of the curve (i.e. the slope of the line tangent to the curve)
- The instantaneous rate of change (often just called the rate of change)

Derivative Rules – sums, differences, products, quotients and composition

- \( \frac{d}{dx} \left( f(x) + g(x) \right) = f'(x) + g'(x) \)
- \( \frac{d}{dx} \left( f(x) - g(x) \right) = f'(x) - g'(x) \)
- \( \frac{d}{dx} \left( cf(x) \right) = cf'(x) \) (where \( c \) is any constant)

- **Product Rule:** \( \frac{d}{dx} \left( f(x) \cdot g(x) \right) = f'(x) \cdot g(x) + f(x) \cdot g'(x) \)

- **Quotient Rule:** \( \frac{d}{dx} \left( \frac{f(x)}{g(x)} \right) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{(g(x))^2} \)

- **Chain Rule:** \( \frac{d}{dx} \left( f(g(x)) \right) = f'(g(x)) \cdot g'(x) \)  
  (Also written as \( \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} \))
Derivative Rules – standard functions

- \( \frac{d}{dx} (c) = 0 \) (where \( c \) is any constant)

- \( \frac{d}{dx} (x^n) = nx^{n-1} \) (where \( n \) is any constant)

- \( \frac{d}{dx} (\log_b x) = \frac{1}{x \ln b} \) (where \( b > 0 \) and \( b \neq 1 \))

- \( \frac{d}{dx} (\ln x) = \frac{1}{x} \)

- \( \frac{d}{dx} (a^x) = a^x \ln a \) (where \( a > 0 \))

- \( \frac{d}{dx} (e^x) = e^x \)

- \( \frac{d}{dx} (\sin x) = \cos x \)

- \( \frac{d}{dx} (\cos x) = - \sin x \)

- \( \frac{d}{dx} (\tan x) = \sec^2 x \)

- \( \frac{d}{dx} (\cot x) = - \csc^2 x \)

- \( \frac{d}{dx} (\sec x) = \sec x \tan x \)

- \( \frac{d}{dx} (\csc x) = - \csc x \cot x \)

- \( \frac{d}{dx} (\arctan x) = \frac{1}{1 + x^2} \)

- \( \frac{d}{dx} (\arcsin x) = \frac{1}{\sqrt{1 - x^2}} \)

- \( \frac{d}{dx} (\arcsec x) = \frac{1}{x \sqrt{x^2 - 1}} \)

- \( \frac{d}{dx} (\sinh x) = \cosh x \) (we may postpone hyperbolic functions until after the third test)

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