Review for Exam 1

1. Find the equation of the tangent line to \( y = \sqrt[3]{\cos(\pi e^x)} \) at the point \((0, -1)\).

2. Use the definition of the derivative to determine the derivative of \( f(x) = \frac{1}{x} \).

3. Calculate the horizontal and vertical asymptotes of \( y = \frac{\sin x}{x} \).

4. Show that there is a sphere of radius \( r \) whose volume \( \left( \frac{4}{3} \pi r^3 \right) \) is one more than its surface area \( (4\pi r^2) \). Be sure to say what theorem you are using, and why it applies (check the hypotheses of the theorem).
5. Calculate the derivative.

\[
\text{(A) } f(x) = x^e \quad \quad \text{(B) } f(\theta) = \frac{\sin \theta}{1+\cos^3 \theta}
\]

\[
\text{(C) } f(x) = \tan(e^{7x})e^{\tan 7x}
\]

6. Given an example where \(f(x)\) and \(g(x)\) have positive derivatives at \(a\) but \(f(x)g(x)\) has a negative derivative at \(a\).

7. In a murder investigation, the temperature of the corpse was 32.5°C at 1:30 PM and 30.3°C an hour later. Normal body temperature is 37.0°C and the temperature of the surroundings was 20.0°C. When did the murder take place?

8. A bacteria culture initially contains 100 cells and grows at a rate proportional to its size. After an hour the population has increased to 450 cells. Find an expression, \(P(t)\), for the number of bacteria after \(t\) hours.
9. True or False?

(a) If \( f(a) \) is defined, then \( f \) is continuous at \( a \).

(b) Every polynomial is continuous.

c) If \( f \) is continuous at \( a \), then \( f \) is differentiable at \( a \).

d) If \( \lim_{x \to a} f(x) \) does not exist and if \( \lim_{x \to a} g(x) \) does not exist, then \( \lim_{x \to a} [f(x) + g(x)] \) can not exist.

e) If \( \lim_{h \to 0} \frac{f(3+h)-f(3)}{h} = 4 \), then \( f \) is differentiable at 3.

(f) The derivative of a rational function is a rational function.