Math 221 Worksheet 7
Wednesday September 19th 2018

Implicit and Logarithmic Differentiation

Instructions. Put the first and last name of everyone in your workgroup at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to show your work and explain your reasoning.

(1) For the following problems, use implicit differentiation or logarithmic differentiation.

(A) For \( w^2q^3 = \sin(w^2) + q^5 \) find \( \frac{dw}{dq} \).

(B) For \( y = (\sec(x))^3 \) find \( \frac{dy}{dx} \).

(2) \( f(t) = \sec(t) \), find \( f''(t) \).

(3) Find the equation of the line tangent to the curve \( x^3 + y^3 = xy^2 + 5 \) at the point (1,2).

(4) Differentiate the following functions. Most functions require you to apply multiple derivative rules to them in the correct order.

(A) \( y = (2z + e^{-z^2})^5 \)

(B) \( y = \ln\left((2x^4 + 3x)^{7/5}\right) \)

(C) \( y = \ln(\ln(x)) \)

(D) \( y = \sec(x)\cot(x)\sin(x)\cos(x)\csc(x) \)
(5) Compute $f'(x)$ given that $f(x) = \tan^{-1}(x^3)$.

(6) Given that $h(t) = \arctan(t^3)$, find its second derivative $h''(t)$.

(7) The hyperbolic sine and cosine are defined by

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \quad \cosh(x) = \frac{e^x + e^{-x}}{2}$$

Show the following identities

$$\frac{d}{dx}(\sinh x) = \cosh x \quad \frac{d}{dx}(\cosh x) = \sinh x$$

(8) Compute the following limits. (Hint: these can be thought of as derivatives).

(a)

$$\lim_{x \to 0} \frac{\tan x^2}{x}$$

(b)

$$\lim_{x \to 0} \frac{e^{3x} - 1}{x}$$