DERIVATIVES WITH THE CHAIN RULE

Instructions. Put the first and last name of everyone in your workgroup at the top of your paper. Everyone is to do their own worksheet but only one from each group is graded with the score shared. Be sure to show your work and explain your reasoning.

1. For the following functions, take the derivative. Use proper derivative notation and equals signs each time.

(A) \( y = -\frac{2}{\sqrt{x + 4}} \)

\[ y' = -2 \left( x + 4 \right)^{-3/2} \]

(B) \( y = \sqrt{-\frac{2}{x} + 4} \)

\[ y' = \frac{1}{2} \left( -2x^{-1} + 4 \right)^{-1/2} \left( -2x^{-2} \right) \]

(C) \( f(x) = \sin^2(x) \)

\[ f'(x) = 2\sin(x)\cos(x) \]

(D) \( h(s) = \sin(s^2) \)

\[ h'(s) = \left[ \cos(s^2) \right] (2s) \]

(E) \( g(t) = \tan(e^t + 2t) \)

\[ g'(t) = \left[ \sec^2(e^t + 2t) \right] (e^t + 2) \]

(F) \( w(u) = e^{\tan(u)} \)

\[ w'(u) = e^{\tan(u)} \cdot \sec^2(u) \]

(G) \( y = (3x^4 + 1)^4(x^3 + 4) \)

\[ y' = (3x^4 + 1)^4 (3x^2) + (x^3 + 4) \left[ 4 (3x^4 + 1)^3 (12x^3) \right] \]

(H) \( g(\alpha) = \frac{\sec(\alpha)}{e^{\alpha^2}} \)

\[ g'(\alpha) = e^{\alpha^2} \left[ \sec(\alpha) \tan(\alpha) \right] - \sec(\alpha) \left[ e^{\alpha^2} \cdot \alpha \right] \]

\[ \left( e^{\alpha^2} \right)^2 \]
2. Find the equation of the tangent line to the graph of \( f(x) = x^2 \sqrt{x^4 - 12} \) at the point \( x = 2 \).

\[
\begin{align*}
 f(x) &= x^2 \left( x^4 - 12 \right)^{\frac{1}{2}} \\
 f'(x) &= x^2 \left[ \frac{1}{2} \left( x^4 - 12 \right)^{-\frac{1}{2}} \cdot 4x^3 \right] + \left( x^4 - 12 \right)^{\frac{1}{2}} \cdot 2x \\
 f'(2) &= \left( 2 \right)^2 \left[ \frac{1}{2} \left( 2^4 - 12 \right)^{-\frac{1}{2}} \cdot 4 \left( 2 \right)^3 \right] + \left( 2^4 - 12 \right)^{\frac{1}{2}} \cdot 4 \\
 &= \frac{64}{2} + 8 = 40
\end{align*}
\]

\[ f(2) = 2^2 \sqrt{2^4 - 12} = 8 \]

\[
\begin{align*}
 y - 8 &= 40 \left( x - 2 \right)
\end{align*}
\]

3. Consider the following table of values of the function \( f \) and \( g \) and their derivatives at various points.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>( f'(x) )</td>
<td>-6</td>
<td>-7</td>
<td>-8</td>
<td>-9</td>
</tr>
<tr>
<td>( g(x) )</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>( g'(x) )</td>
<td>2/7</td>
<td>3/7</td>
<td>4/7</td>
<td>5/7</td>
</tr>
</tbody>
</table>

Below is another notation for derivative. Find the derivatives of the compositions below, using the chain rule and the table above.

a) \( D_x(f(g(x))) \) at \( x = 1 \)
\[
\begin{align*}
 f'(g(1)) \cdot g'(1) &= f'(2) \left( \frac{2}{7} \right) \\
 &= -7 \left( \frac{2}{7} \right) = \boxed{-2}
\end{align*}
\]

b) \( D_x(f(g(x))) \) at \( x = 2 \)
\[
\begin{align*}
 f'(g(2)) \cdot g'(2) &= f'(3) \left( \frac{3}{7} \right) = -8 \left( \frac{3}{7} \right) = \boxed{-\frac{24}{7}}
\end{align*}
\]

c) \( D_x(g(f(x))) \) at \( x = 1 \)
\[
\begin{align*}
 g'(f(1)) \cdot f'(1) &= g'(2) \left( -6 \right) = 3/7 \left( -6 \right) = \boxed{-\frac{18}{7}}
\end{align*}
\]

d) \( D_x(g(g(x))) \) at \( x = 2 \)
\[
\begin{align*}
 g'(g(2)) \cdot g'(2) &= g'(3) \left( \frac{3}{7} \right) = \left( \frac{4}{7} \right) \left( \frac{3}{7} \right) = \boxed{\frac{12}{49}}
\end{align*}
\]